Part 1

In this part, you will calculate the location of the Lagrange Point $L1$ between the Earth and the Sun. This is a special location in the Solar System where the gravitational force from the Earth and the Sun equals the centrifugal force felt by a third body. The exact location of $L1$ can only be calculated numerically. To do this, implement a function $\text{getL1}(M_1, M_2)$ that uses Newton’s method to solve the following equation for $r$:

$$0 = -\frac{M_1}{(R - r)^2} + \frac{M_2}{r^2} + \frac{M_1}{R^2} - \frac{r (M_1 + M_2)}{R^3}$$

(1)

where w.l.o.g. you may set $R = 1$. For Newton’s methods, you need the derivate of this equation. Calculate the derivative analytically. As an initial guess for Newton’s methods you may use $r_0 = 0.1$. Once your solution has converged to almost machine precision, return the approximate value of $r$.

$M_1$ and $M_2$ are the masses of the two massive bodies, i.e. the Sun and the Earth. Your function should work for any value of $M_1$ and $M_2$, but you may want to test your code with the values for the Sun and the Earth, $M_1 = 1.0, M_2 = 3.0 \cdot 10^{-6}$, for which the solution is $r \approx 0.00996655277$. For the physicists among you: we’re working in units where $G = 1$. 
Part 2

1. Go to NASA's Horizon website at http://ssd.jpl.nasa.gov/?horizons and download the trajectory of both the Voyager 1 spacecraft and Saturn. To do this, click on 'web interface', then select the following options:

   Ephemeris Type: Vector Table
   Target Body: Voyager 1
   Coordinate Origin: Solar System Barycenter
   Time Span: Start=1980-11-10, Stop=1980-11-30, Step=1 d
   Table Settings: defaults
   Display/Output: plain text

Repeat the above process, with the exact same options except for the target which you should change to Saturn Barycenter. Save the complete output as two text files, voyager1.txt and saturn.txt.

The data format is to be understood as follows. The important part starts after the sequence $$SOE$$ and ends before the sequence $$EOE$$. In between those statements, four lines correspond to one data point. For example:

$$2444553.500000000 = \text{A.D. 1980-Nov-10 00:00:00.0000 TDB}$$
$$-9.464721239400E+00 -3.3984345041628E-01 3.8004817290153E-01$$
$$-8.89603480199E-03 -7.82494864272E-03 4.9991519987601E-03$$
$$-5.7442664767881E-02 9.4783975728853E+00 9.1551625552845E-03$$

Here, the underlined quantities are the time and the x, y and z coordinates of the object. You may ignore all other quantities. You also don’t have to worry about the units. For the physicists: the units of time and length in this data file are Julian days and astronomical units, respectively.

2. Write a python program to read in the two data files you just downloaded. You may want to search for the sequences $$SOE$$, then read in four lines at a time for every data point and stop when you encounter the sequence $$EOE$$. If you don’t know how to do this, ask for help in the tutorial and we will show you how to do this part. You should create the following three variables:

   d.t: A list of floats that correspond to the time column in your data files (note that this is the same for both files).
   d.sat.xyz: A list of lists, containing the Cartesian xyz coordinates of Saturn as floats, e.g. the list should have the form: [[-9.4721E+00, -3.848E-01, 3.813E-01], [-9.462E+00, -3.334E-01, 3.723E-01], ...].
   d.voy.xyz: Same as above but for the coordinates of Voyager 1.

3. Create a function to interpolate the distance between Saturn and Voyager 1 in-between data points. Name the function getDistance(t). It should return the interpolated distance at the time t. To do this, first calculate the distance between the two bodies at the two nearest datapoints, then use the linear interpolation method to approximate the distance at time t. You may assume that t is in the range where you have data points.
4. Implement a function `getTime(d)` which uses the bisection method and your function `getDistance(t)` from above to find when Voyager 1 was exactly a distance $d$ away from Saturn. Stop the bisection when you have narrowed down the time interval to within 0.0001 days. You may assume that the answer is always in the range where you have data points.

You can test your function by searching for the time when Voyager 1 was 0.03542 AU away from Saturn. The answer should be $\approx 2444560.23090$ JD, which is Nov. 16, 1980. The following image was taken at that time at a distance of 5.3 million kilometres.

![Figure 1: A picture taken by Voyager 1 looking back at Saturn on Nov. 16, 1980, four days after the spacecraft flew past the planet, to observe the appearance of Saturn and its rings from this unique perspective. A few of the spoke-like ring features discovered by Voyager appear in the rings as bright patches in this image, taken at a distance of 5.3 million kilometres (3.3 million miles) from the planet. Saturn’s shadow falls upon the rings, and the bright Saturn crescent is seen through all but the densest portion of the rings. From Saturn, Voyager 1 is on a trajectory taking the spacecraft out of the ecliptic plane, away from the Sun and eventually out of the Solar System. Image credit: NASA/JPL](image)