Classical Mechanics, PHYB54
Problem Set 7

Professor Hanno Rein
TA Rejean Leblanc

Due: Monday, March 27th 2017, 4pm

Note: Assignments can be hand-written, but illegible answers will not be marked. Clearly indicate your final answers.

Problem 7.1

A mass m is suspended from the ceiling with a spring with constant k. Attached to this mass is another spring with constant $4k$ and a mass $7m$ on the other end. Assume only vertical motion. Use displacement from equilibrium of the two masses as a coordinate system. Write down the matrix form of the equations of motion $\mathbf{M}\ddot{y} = -\mathbf{K}y$. That is, find matrices $\mathbf{M}$ and $\mathbf{K}$ for this system.

Problem 7.2

A long molecule such as a polymer can be thought of as a system of tense strings and masses which oscillate. Imagine a system of three pieces of string length L with two masses m in between them and a tension T holding them. The far ends of the strings are pinned down for simplicity, and we assume only transverse oscillations. Hint: for small displacements, L is approximately constant, we can say the potential energy in the string is $T \cdot d$ where T is the tension in the string, and d is the displacement along the tension.

(a) What is the Lagrangian of this system?

(b) What are the modes and frequencies of this system?

(c) If you were to write a code to numerically simulate this system (concerning only the transverse motion of the molecule as it is pinned down), how would you approach this? Explain the general idea using the equations you derived. You don't need to write any code but you may refer to specific packages/modules.

Problem 7.3

For a general second order equation of the form

$$p(t)\,\ddot{x}(t) + q(t)\,\dot{x}(t) + r(t)\,x(t) = 0$$

give a detailed proof of the superposition principle. Show how this fails for a nonlinear equation such as

$$p(t)\,\ddot{x}(t) + q(t)\,\dot{x}(t)^2 + r(t)\,x(t)^{\frac{1}{2}} = 0.$$