Classical Mechanics, PHYB54
Problem Set 5

Professor Hanno Rein

Due: Monday, March 6th 2017, 4pm

Note: For this assignment, you must hand in your answers as well as a printout of your source code and the plots you made. Make sure the plots have labels on the axes and a high enough sampling rate to produce smooth results. This assignment counts twice as much as each of the first four assignments.

Problem 5.1

Your task is to write a computer program to integrate the differential equation of a forced damped harmonic oscillator. The equation of motion is a second order differential equation:

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = f.$$

Will will assume a forcing of the form

$$f(t) = f_0 \sin(\omega t).$$

Most numerical integration packages cannot solve second order differential equations. Therefore, rewrite the second order differential equation as two coupled first order differential equations in the variables x and v, i.e.:

$$\begin{pmatrix} \dot{x} \\ \ddot{x} \end{pmatrix} = \begin{pmatrix} x \\ v \end{pmatrix}^{\cdot} = \dots$$

The right hand side should only contain the variables x and v, i.e. not $\dot{x}$.

Problem 5.2

Write a computer program to integrate the two first-order differential equations from 5.1. You may use a package such as `numpy` or `scipy` to integrate the coupled differential equations. However, you have to implement the function that calculates the derivate yourself. Test your program with the following parameters:

$$\beta = 0.1 \qquad \omega_0 = 1 \qquad f_0 = 2.3 \qquad \omega = 1.7$$

and the following initial conditions:

$$t_0 = 0 \qquad x(t_0) = 0 \qquad v(t_0) = 0$$

After a few oscillation periods, you should see a stable oscillation with an amplitude of ~ 1.25.

(a) Plot both the position and velocity as a function of time in the interval from $t = 0$ to $t = 50$.

(b) Create the same plot again, but change $x(t_0) = 9$. Describe qualitatively which aspects of the solution change and which aspects stay the same.

(c) Create the same plot again. This time use $x(t_0) = 0$ and $w = 1$. Describe qualitatively which aspects of the solution change and **why**.