Migration of propellers in Saturn's rings

Hanno Rein @ ISIMA 2011 KIAA Beijing
Symplectic integrators

Observations

Possible explanations
Numerical Integrators

- We want to integrate the equations of motions of a particle
  \[ \dot{x} = v \]
  \[ \dot{v} = a(x, v) \]

- For example, gravitational potential
  \[ a(x) = -\nabla \Phi(x) \]

- In physics, these can usually be derived from a Hamiltonian
  \[ H = \frac{1}{2} p^2 + \Phi(x) \]

- Symmetries of the Hamiltonian correspond to conserved quantities
Numerical Integrators

- Discretization
  \[
  \dot{x} = v \\
  \dot{v} = a(x, v)
  \]
  \[
  \Delta x = v \Delta t \\
  \Delta v = a(x, v) \Delta t
  \]

- Hamiltonian
  \[
  H = \frac{1}{2} p^2 + \Phi(x)
  \]
  \[
  ?
  \]

- The system is governed by a 'discretized Hamiltonian', if and only if the integration scheme is symplectic.

- Why does it matter?
Symplectic vs non symplectic integrators
Mixed variable integrators

- So far: symplectic integrators are great.
- How can it be even better?
- We can split the Hamiltonian:

\[ H = H_0 + \epsilon H_{\text{pert}} \]

Integrate particle exactly with dominant Hamiltonian
Integrate particle exactly under perturbation Hamiltonian

- Switch back and forth between different Hamiltonians
- Often uses different variables for different parts
- Then:

\[ \text{Error} = \epsilon (\Delta t)^{p+1} [H_0, H_{\text{pert}}] \]
Example: Leap-Frog

\[ H = \frac{1}{2} p^2 + \Phi(x) \]

\[ \text{Drift} \quad \text{Kick} \]

1/2 Drift → Kick → 1/2 Drift
Example: SWIFT/MERCURY

\[ H = \frac{1}{2} p^2 + \Phi_{\text{Kepler}}(x) + \Phi_{\text{Other}}(x) \]
Example: Symplectic Epicycle Integrator

\[
H = \frac{1}{2}p^2 + \Omega(p \times r)e_z + \frac{1}{2}\Omega^2 \left[r^2 - 3(r \cdot e_x)^2\right] + \Phi(r)
\]

1/2 Kick \rightarrow Epicycle \rightarrow 1/2 Kick
10 Orders of magnitude better!

- non-symplectic
- symplectic
- mixed variable, symplectic

Rein & Tremaine 2011
Conclusions
Part I
Conclusions

Symplectic integrators

Hamiltonian systems exhibit large number of symmetries
These are usually lost in standard integrators (RK)
Symplectic integrators keep symmetries (might be modify slightly)
No secular drift
Excellent performance
Worth thinking about!
Symplectic integrators

Observations

Possible explanations
Propeller structures in A-ring

Porco et al. 2007, Sremcevic et al. 2007, Tiscareno et al. 2006
Observational evidence of non-Keplerian motion

Figure 4. Observed longitude of the propeller "Blériot" over 5 years with a linear trend ($\sigma < k_{\text{km}}$) subtracted. Only data points with measurement errors $\sigma < k_{\text{km}}$ are shown. Error bars are given but in many cases are smaller than the plotting symbol. Panel a shows all the data while panels b, c, and d contain subsets of the data shown in greater detail. The residuals to the linear trend (horizontal dotted line) are less than $\pm k_{\text{km}}$ but are clearly not randomly distributed. The dotted line indicates a linear plus sinusoidal fit to all the data with an amplitude of $k_{\text{km}}$ and a period of $nq_{\text{ksy}}$ The solid lines indicate piecewise quadratic fits corresponding to a constant drift in semimajor axis in particular, the data from mid-2006 to early 2009 (panel c) are fit by a linear trend with a constant acceleration of $k_{\text{km}}$ while the data from late 2009 to early 2010 (panel d) are fit by a linear trend with a constant acceleration of $k_{\text{km}}$.

Table 1

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<thead>
<tr>
<th>Nickname</th>
<th>Longitude Rms deviation $n_{\text{f}}$</th>
<th>Epoch</th>
<th>Time interval in km in longitude $n_{\text{f}}$</th>
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Formal error estimates shown in parentheses for the last digit are for the best fit linear trend in longitude. They are much smaller than the rms deviations in longitude given in the right-hand column.

Epoch is January at UTC. All orbit fits assume $e$ and $i$.

Not including images of insufficient quality to include in the orbit fit exclusively proven that giant propellers are missing in the Propeller Belt. Even the largest propellers observed in the Propeller Belt have $\Delta r < w_{\text{km}}$.z km nTiscareno et al. 2010 while nearly all observed trans-Encke propellers have $\Delta r$ larger than this value in Figure 3.

THE ORBITAL EVOLUTION OF "BLÉRIOT"

At least 50 propellers have been seen at multiple widely separated instances, but "Blériot" is of particular interest as the largest and most frequently detected. It has appeared in more than one hundred separate Cassini ISS images spanning a period of four years, and was serendipitously detected once in a stellar occultation observed by the Cassini UVIS instrument. Analysis of the orbit of "Blériot" confirms that it is both long-lived and reasonably well-characterized by a keplerian path. As Figure 4 shows, a linear fit to the longitude with time (corresponding to a circular orbit) results in residuals of $\pm z_{\text{km}}$ longitude. Hows Tiscareno et al. 2010
Longitude residual

Mean motion [rad/s]
\[ n = \sqrt{\frac{GM}{a^3}} \]

Mean longitude [rad]
\[ \lambda = n t \]

\[ \lambda(t) - \lambda_0(t) = \int_0^t (n_0 + n'(t')) \, dt' - \int_0^t n_0 \, dt' + n_0 t \]
Keplerian rotation: linear

\[ n'(t) = \text{const} \]

\[ \lambda(t) - \lambda_0(t) = \int_0^t (n_0 + n'(t')) \, dt' - \int_0^t n_0 \, dt' = n_0 t + n' t - n_0 t = n' t \]
Constant migration rate: quadratic

\[ n'(t) = \text{const} \cdot t \]

\[
\lambda(t) - \lambda_0(t) \\
= \int_0^t (n_0 + n'(t')) \, dt' \\
- \int_0^t n_0 \, dt' \\
= \frac{1}{2} \text{const} \cdot t^2
\]
Resonance: sine-curve

\[ n'(t) = \cos(t) \]

\[ \lambda(t) - \lambda_0(t) \]
\[ = \int_0^t (n_0 + n'(t')) \, dt' \]
\[ - \int_0^t n_0 \, dt' \]
\[ = \sin(t) \]
Random walk

\[ n'(t) = \int_0^t F(t') \, dt' \]

\[ \langle F(t) \rangle = 0 \]

\[ \langle F(t) F(t + \Delta t) \rangle = \langle F^2 \rangle e^{-\Delta t/\tau_c} \]

\[ \langle (\lambda(t) - \lambda_0(t))^2 \rangle \]

\[ = \int \int \int \int_0^{t,t',t,t'''} F(t'') F(t''') \, dt''' \, dt'' \, dt' \]

\[ = \langle F^2 \rangle \left( -2\tau^4 + \left( 2\tau^3 t + 2\tau^4 + \tau^2 t^2 \right) e^{-t/\tau} + \frac{1}{3} \tau t^3 \right) \]
Random walk

\[ n'(t) = \int_0^t F(t') \, dt' \]

\[
|\lambda(t) - \lambda_0(t)| = \sqrt{\frac{\langle F^2 \rangle}{\tau}} t^{3/2}
\]

On average!
Observational evidence of non-Keplerian motion

Figure 4. Observed longitude of the propeller "Blériot" over years with a linear trend with data points with measurement errors $\sigma < k$. Error bars are given but in many cases are smaller than the plotting symbol. Panel cad shows all the data while panels cbdg ccdg and cdd contain subsets of the data shown in greater detail. The residuals to the linear trend (horizontal dotted line) are less than $\pm z$ but are clearly not randomly distributed. The dotted line indicates a linear plus sinusoidal fit to all data with an amplitude of $k$ and a period of $n$.

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c

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THE ORBITAL EVOLUTION OF "BLÉRIOT"

At least $w$ propellers have been seen at multiple widely separated instances but "Blériot" is of particular interest as the largest and most frequently detected. It has appeared in more than one hundred separate Cassini ISS images spanning a period of four years and was serendipitously detected once in a stellar occultation observed by the Cassini UVIS instrument. Analysis of the orbit of "Blériot" confirms that it is both long-lived and reasonably well-characterized by a keplerian path. As Figure 4 shows, a linear fit to the longitude with time (corresponding to a circular orbit) results in residuals of $\pm z$ longitude. Hows Tiscareno et al. 2010
Symplectic integrators

Observations

Possible explanations
Resonance with a moon

**PRO**
- Produces sine-shaped residual longitude
- Amplitude is a free parameter

**CONTRA**
- No resonance found
- Cannot fully explain shape of observations
- Other moonlets seem to migrate as well
Modified Type I Migration

- Due to curvature (would be zero in shearing sheet)
- Similar to planetary migration in a gas disk
- No gas pressure
- Migration rate can be calculated analytically

\[
\frac{dr_m}{dt} = -35.6 \frac{\Sigma r_m^2}{M} \left( \frac{m}{M} \right)^{1/3} r_m \Omega.
\]
Modified Type I Migration

PRO

• Robust

• Would be a direct observation of type I migration

CONTRA

• Tiny migration rate ~20 cm/year

• Cannot explain shape of observations
Frog resonance

Saturn

- Propeller
- Co-orbital mass

Pan & Chiang 2010
Frog resonance

**PRO**
- Predicts largest period very well
- Amplitude is a free parameter

**CONTRA**
- Unclear if density distribution is like in the toy model (see Eugene's ISIMA project)
- Cannot fully explain shape of observations
Random Walk

Rein & Papaloizou 2010, Crida et al 2010
Two different approaches

Analytic model
Describing evolution in a statistical manner
Partly based on Rein & Papaloizou 2009

\[ \Delta a = \sqrt{4 \frac{D t}{n^2}} \]

\[ \Delta e = \sqrt{2.5 \frac{\gamma D t}{n^2 a^2}} \]

N-body simulations
Measuring random forces or integrating moonlet directly
Crida et al 2010, Rein & Papaloizou 2010
Effects contributing to the eccentricity evolution

- Laminar collisions
- Particles colliding
- Laminar horseshoe
- Laminar circulating
- Particles circulating
- Clumps circulating

Damping
Excitation

Equilibrium eccentricity

Rein & Papaloizou 2010, Crida et al 2010
...semi-major axis evolution

- Particles colliding
- Particles horseshoe
- Particles circulating
- Clumps circulating
- Damping
- Excitation
- Random walk in semi-major axis
  +Net “Type I” migration

Rein & Papaloizou 2010, Crida et al 2010
PRO
• Can explain the shape of the observations very well

CONTRA
• Has only been tested numerically for small moonlets (ISIMA project with Shangfei)
• No metric to test how good it matches the observations
Hybrid Type I Migration / Stochastic Kicks

\[ \delta \Sigma / \Sigma \sim 3\% \]
Hybrid Type I Migration / Stochastic Kicks

**PRO**
- Can explain all observations very well

**CONTRA**
- Many free parameters: surface density profile, kicks
- Needs large kicks (maybe not)

Tiscareno (in prep)
Figure 4. Observed longitude of the propeller "Blériot" over 4 years with a linear trend after subtracting only data points with measurement errors $\sigma < k$. Error bars are given but in many cases are smaller than the plotting symbol. Panel cad shows all the data while panels cbdg ccdg and cdd contain subsets of the data shown in greater detail. The residuals to the linear trend (horizontal dotted line) are less than $\pm knk kmjyr$ but are clearly not randomly distributed. The dotted line indicates a linear/hinusoidal fit to all the data with an amplitude of $k$ and a period of $nqri$. The solid lines indicate piecewise quadratic fits corresponding to a constant drift in semimajor axis in particular the data from mid $kmkkq$ to early $hmkk$ (panel cd) are fit by a linear trend with a constant acceleration of $hkikktq$ while the data from late $hmkk$ to early $hmkk$ (panel dd) are fit by a linear trend with a constant acceleration of $fkikknm$.

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Conclusions
Part II
Conclusions

Moonlets in Saturn’s rings
- Small scale version of the proto-planetary disc
- Dynamical evolution can be directly observed
- 5 different explanations
- Might lead to independent age estimate of the ring system

Modified Type I Migration
Random Walk
Frog Resonance
Hybrid Migration/Random Walk
Resonance with a moon
Thank you!