Formation and stability of planetary systems

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Collaborators

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1. Solar System
2. WHFast
3. Reproducibility
4. JANUS
5. Machine Learning
1. The Solar System
Gravity

\[ \ddot{r}_i = \sum_{\substack{j=1 \atop j \neq i}}^{N} m_j \frac{r_j - r_i}{|r_j - r_i|^3} \]
### History of secular perturbation theory

<table>
<thead>
<tr>
<th>$k$</th>
<th>Lagrange (1774)</th>
<th>Laskar et al., 2004</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_1$</td>
<td>5.980</td>
<td>5.59</td>
</tr>
<tr>
<td>$s_2$</td>
<td>6.311</td>
<td>7.05</td>
</tr>
<tr>
<td>$s_3$</td>
<td>19.798</td>
<td>18.850</td>
</tr>
<tr>
<td>$s_4$</td>
<td>18.308</td>
<td>17.755</td>
</tr>
<tr>
<td>$s_5$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$s_6$</td>
<td>25.337</td>
<td>26.347</td>
</tr>
</tbody>
</table>

Laskar 2012
Previous direct numerical simulations

LONGSTOP (1982)
- Outer planets only
- No instability

Digital Orrery (1988)
- Outer planets only, 800 Myr
- Pluto is chaotic

Laskar (1989)
- All planets, averaged equations
- Earth is chaotic on a 100 Myr timescale

Laskar (2009)
- All planets, full equations
- Collisions between terrestrial planets possible
2. WHFast
Mixed Variable Symplectic Integrator

\[ H = \frac{1}{2} p^2 + \Phi_{\text{Sun}}(q) + \Phi_{\text{Other}}(q) \]

Particularly good if

\[ \frac{1}{2} p^2 + \Phi_{\text{Sun}}(q) \gg \Phi_{\text{Other}}(q) \]
Symplectic integrators

Symplectic integrator

Non-symplectic integrator
- 2nd order symplectic integrator with up to 11th order symplectic correctors
- Extremely efficient implementation of a Wisdom-Holman Map (WHM)
- WHFast can integrate variational equations
- Bias free implementation
2-body results

Rein & Tamayo 2015
2-body results

WH

WHFast

Rein & Tamayo 2015
3. Exact Reproducibility
Reproducible Experiments

Exact reproducibility

- N-body simulations are experiments on a computer
- They do not represent the real physical system
- Simplifications lead to a controllable experiment
- Yet none of the published results are reproducible

Why does it matter?

- Reproducibility is the hallmark of good science
- Non-reproducibility raises bar for follow-up investigations
- Wasted resources, e.g. 6.2 million CPU hours by Laskar 2009
Reproducible Experiments

Reasons for non-reproducible results

- Source code not available
- Initial conditions not available
- Machine dependent software

**REBOUND + Simulation Archive**

- Open Source
- Machine independent
- Exactly reproducible
- Easy to share simulations with the SimulationArchive
demo / rebound

rein010.utsc.utoronto.ca
4. Janus
5. Machine Learning
3668
x
1,000,000,000,000
x
10,000
Migration saves Trappist-1

Tamayo, Rein, Petrovich & Murray 2017
Stability implies small amount of turbulence

Tamayo, Rein, Petrovich & Murray 2017
Idea

- Long term integrations of many systems still hard to do or even impossible
- Train a ML algorithm on short integrations to predict long term outcome
Machine Learning to Predict Stability

Tamayo, Rein et al (2016)
Accuracy of Machine Learning Algorithm

Tamayo, Rein et al (2016)
Work by Dan Tamayo. Watch out for a new paper!
How to generate training data
Thank you!

Try out REBOUND!

github.com/hannorein/rebound