1. Multi-planetary systems
2. Saturn's Rings
3. The collisional N-body code REBOUND

Hanno Rein @ TITech, Tokyo, March 2012
Migration in a non-turbulent disc
Planet formation

Image credit: NASA/JPL-Caltech
Migration - Type I

- Low mass planets
- No gap opening in disc
- Migration rate is fast
- Depends strongly on thermodynamics of the disc
Migration - Type II

- Massive planets (typically bigger than Saturn)
- Opens a (clear) gap
- Migration rate is slow
- Follows viscous evolution of the disc
Gap opening criteria

\[
\frac{3}{4} \frac{H}{R_{\text{Hill}}} + \frac{50 M_*}{M_p \mathcal{R}} \leq 1
\]
Migration - Type III

- Massive disc
- Intermediate planet mass
- Tries to open gap
- Very fast, few orbital timescales
planet + disc = migration
Resonance capture
2 planets + migration = resonance
HD 45364
HD45364

Observations vs Correia et al

Radial velocity [m/s]

JD-2400000 [days]

HD 45364 b
HD 45364 c

Pluto
Mercury
Mars
Venus
Earth
Neptune
Uranus
Saturn
Jupiter

Formation scenario for HD45364

- Two migrating planets
- Infinite number of resonances
- Migration speed is crucial
- Resonance width and libration period define critical migration rate

\[ \frac{P_2}{P_1} = 2 \]

\[ \frac{\Delta \omega}{\omega} \approx \frac{3}{2} \]

\[ \tau \approx 810 \text{ yrs} \]

\[ \frac{P_2}{P_1} \approx 1.4 \]

\[ \frac{P_2}{P_1} \approx 2.8 \]

\[ \frac{P_2}{P_1} \approx 2.4 \]

\[ \frac{P_2}{P_1} \approx 2.2 \]

\[ \frac{P_2}{P_1} \approx 2.0 \]

\[ \frac{P_2}{P_1} \approx 1.8 \]

\[ \frac{P_2}{P_1} \approx 1.6 \]

\[ \frac{P_2}{P_1} \approx 1.4 \]
The outer planet's gap, while the inner planet remains sembedded at the end of the type III migration phase. The outer planet establishes centricities (e.g. type I migration rates (e.g. commensurability). Although it is difficult to rule out such possibilities entirely, we note that the cores would be expected to be in the super earth mass range, where in general closer 3:2 commensurability. Although it is difficult to rule out such possibilities entirely, we note that the cores would be expected to be in the super earth mass range, where in general closer 3:2 commensurability. Although it is difficult to rule out such possibilities entirely, we note that the cores would be expected to be in the super earth mass range, where in general closer 3:2 commensurability.

An issue is whether the embedded inner planet is in a rapid accretion phase. The total planet mass depends about equal (<em>P</em>) occurs when the core and envelope mass are about equal (<em>P</em>), but even 3:2 are found for typical <em>P</em>. After 150 orbits (<em>P</em>), the outer planet approached the inner planet more closely than (<em>P</em>), and the circumplanetary flow. When these allow the planet to have a significant convective envelope, the transition to rapid accretion becomes possible. The semi-major axes (<em>a</em>) of the two planets plotted a safety function of time in the bottom panel of the simulation.
Formation scenario for HD45364

Massive disc (5 times MMSN)

• Short, rapid Type III migration
• Passage of 2:1 resonance
• Capture into 3:2 resonance

Large scale-height (0.07)

• Slow Type I migration once in resonance
• Resonance is stable
• Consistent with radiation hydrodynamics

Rein, Papaloizou & Kley 2010
Formation scenario leads to a better ‘fit’

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Correia et al. (2009)</th>
<th>Simulation F5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M \sin i$</td>
<td>[M$_{\text{Jup}}$]</td>
<td>0.1872 0.6579</td>
<td>0.1872 0.6579</td>
</tr>
<tr>
<td>$M_*$</td>
<td>[M$_{\odot}$]</td>
<td>0.82</td>
<td>0.82</td>
</tr>
<tr>
<td>$a$</td>
<td>[AU]</td>
<td>0.6813 0.8972</td>
<td>0.6804 0.8994</td>
</tr>
<tr>
<td>$e$</td>
<td>[deg]</td>
<td>0.17 ± 0.02 0.097 ± 0.012</td>
<td>0.036 0.017</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>[deg]</td>
<td>105.8 ± 1.4 269.5 ± 0.6</td>
<td>352.5 153.9</td>
</tr>
<tr>
<td>$\varpi^a$</td>
<td>[deg]</td>
<td>162.6 ± 6.3 7.4 ± 4.3</td>
<td>87.9 292.2</td>
</tr>
<tr>
<td>$\sqrt{\chi^2}$</td>
<td></td>
<td>2.79</td>
<td>2.76$^b$ (3.51)</td>
</tr>
<tr>
<td>Date</td>
<td>[JD]</td>
<td>2453500</td>
<td>2453500</td>
</tr>
</tbody>
</table>

Rein, Papaloizou & Kley 2010
Migration in a turbulent disc
Turbulent disc

- Angular momentum transport
- Magnetorotational instability (MRI)
- Density perturbations interact gravitationally with planets
- Stochastic forces lead to random walk
- Large uncertainties in strength of forces

Animation from Nelson & Papaloizou 2004
Random walk

pericenter

eccentricity

semi-major axis

time

Analytic Model
Correction factors are important

\[(\Delta a)^2 = 4 \frac{Dt}{n^2}\]

\[(\Delta \omega)^2 = 2.5 \frac{\gamma Dt}{e^2 \frac{n^2 a^2}{}}\]

\[(\Delta e)^2 = 2.5 \frac{\gamma Dt}{n^2 a^2}\]
Multi-planetary systems in mean motion resonance

- Stability of multi-planetary systems depends strongly on diffusion coefficient
- Most planetary systems are stable for entire disc lifetime

Rein & Papaloizou 2009

GJ876

Earth
HD128311 has a very peculiar libration pattern.

Can not be reproduced by convergent migration alone.

Turbulence can explain it.

More multi-planetary systems needed for statistical argument.
Migration scenarios can explain the dynamical configuration of many systems in amazing detail.
HD200964
The impossible system?
Two massive planets
1.8 $M_{\text{Jup}}$ and 0.9 $M_{\text{Jup}}$

Period ratio either
3:2 or 4:3

Another similar system, to be announced soon

How common is 4:3?

Formation?
Standard disc migration doesn't work

**observed masses**

**reduced masses**
Stability of HD200964
Hydrodynamical simulations
Hydrodynamical simulations III

Rein, Payne, Vera & Ford (2012 in prep)
Scattering of embryos

Rein, Payne, Vera & Ford (2012 in prep)
HD200964

• In situ formation?
• Main accretion while in 4:3 resonance?
• Planet planet scattering?
• A third planet?
• Observers screwed up?
There is still a lot that we do not understand
Moonlets in Saturn's Rings
Propeller structures in A-ring

Porco et al. 2007, Sremcevic et al. 2007, Tiscareno et al. 2006
Longitude residual

Mean motion [rad/s]

\[ n = \sqrt{\frac{GM}{a^3}} \]

Mean longitude [rad]

\[ \lambda = nt \]

\[ \lambda(t) - \lambda_0(t) = \int_0^t (n_0 + n'(t')) \, dt' - \int_0^t n_0 \, dt' \]
Observational evidence of non-Keplerian motion

Figure 4. Observed longitude of the propeller "Blériot" over n years with a linear trend and sinusoidal fit to all the data, with residuals less than ±km but clearly not randomly distributed.

Table 1: Orbit fits for trans-Encke propellers

<table>
<thead>
<tr>
<th>Nickname</th>
<th>Longitude Rms deviation</th>
</tr>
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<tbody>
<tr>
<td>Earhart</td>
<td></td>
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<tr>
<td>Post</td>
<td></td>
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<tr>
<td>Curtiss</td>
<td></td>
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<tr>
<td>Lindbergh</td>
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<tr>
<td>Wright</td>
<td></td>
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<td>Kingsford Smith</td>
<td></td>
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<tr>
<td>Hinkler</td>
<td></td>
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<tr>
<td>Santos Dumont</td>
<td></td>
</tr>
<tr>
<td>Richthofen</td>
<td></td>
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<tr>
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Formal error estimates shown in parentheses for the last digits are for the best-fit linear trend in longitude. They are much smaller than the rms deviations in longitude given in the right-hand column.

Epoch is liiq January at ltii UTC aJD qrlripgibg All orbit fits assume w i and i w ig

Not including images of insufficient quality to include in the orbit fit exclusively proven that giant propellers are missing in the Propeller Beltss Even the largest propellers observed in the Propeller Belts have r<v km mTiscareno et als wuu8nq while nearly all observed trans-Encke propellers have r larger than this value mFigs wns.

THE ORBITAL EVOLUTION OF "BL´ERIOT"

At least vv propellers have been seen at multiple widely-separated instances but "Blériot" is of particular interest as the largest and most frequently detected. It has appeared in more than one hundred separate Cassini ISS images spanning a period of four years and was serendipitously detected once in a stellar occultation observed by the Cassini UVIS instrument mColwell et als wuu8q wuvuns.

Analysis of the orbit of "Blériot" confirms that it is both long-lived and reasonably well-characterized by a keplerian path. As Figs z shows, a linear fit to the longitude with time mcorresponding to a circular orbitn results in residuals of ±km musvy longitudens Howr

Tiscareno et al. 2010
Random walk

Analytic model
Describing evolution in a statistical manner
Partly based on Rein & Papaloizou 2009

\[
\Delta a = \sqrt{4 \frac{Dt}{n^2}}
\]

\[
\Delta e = \sqrt{2.5 \frac{\gamma Dt}{n^2 a^2}}
\]

N-body simulations
Measuring random forces or integrating moonlet directly
Crida et al 2010, Rein & Papaloizou 2010
Random walk

REBOUND code, Rein & Papaloizou 2010, Crida et al 2010
Work in progress: a statistical measure

Figure 4. Observed longitude of the propeller "Blériot" over n years with a linear trend bpkphqrksmlsidayc subtracted o only data points with measurement errors <jжj. Error bars bkgsigmac are givenf but in many cases are smaller than the plotting symbol. Panel bac shows all the dataf while panels bbcf bccf and bdc contain subsets of the data shown in greater detail. The residuals to the linear trend bhorizontal dotted linec are less than ±jжj kmf but are clearly not randomly distributed. The dotted line indicates a lineargplusgsinusoidal fit to all the dataf with an amplitude of jжj иjжж and a period of mhpr yrh. The solid lines indicate piecewise quadratic fitsf corresponding to a constant drift in semimajor axisu in particularf the data from midgljjp to earlygljjq bpanel cc are fit by a linear trend with a constant acceleration of gjhjjsp

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-300
-200
-100
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100
200
300
-300
-200
-100
0
100
200
300
0
0.5
1
1.5
2
2.5
3
3.5
4
0
0.5
1
1.5
2
2.5
3
3.5
4
0
0.05
0.10
0.00
0.05
0.10
0.15
2005
2006
2007
2008
2009
2010
Saturn's rings = small scale version of a proto-planetary disc
REBOUND
A new open source collisional N-body code
Numerical Integrators

• We want to integrate the equations of motions of a particle

\[ \dot{x} = v \]
\[ \dot{v} = a(x, v) \]

• For example, gravitational potential

\[ a(x) = -\nabla \Phi(x) \]

• In physics, these can usually be derived from a Hamiltonian

\[ H = \frac{1}{2}p^2 + \Phi(x) \]

• Symmetries of the Hamiltonian correspond to conserved quantities
Numerical Integrators

• Discretization

\[
\begin{align*}
\dot{x} &= v \\
\dot{v} &= a(x, v)
\end{align*}
\]

\[
\begin{align*}
\Delta x &= v \Delta t \\
\Delta v &= a(x, v) \Delta t
\end{align*}
\]

• Hamiltonian

\[
H = \frac{1}{2}p^2 + \Phi(x)
\]

• The system is governed by a 'discretized Hamiltonian', if and only if the integration scheme is symplectic.

• Why does it matter?
Symplectic vs non symplectic integrators
Mixed variable integrators

• So far: symplectic integrators are great.
• How can it be even better?
• We can split the Hamiltonian:

\[ H = H_0 + \epsilon H_{\text{pert}} \]

Integrate particle exactly with dominant Hamiltonian
Integrate particle exactly under perturbation Hamiltonian

• Switch back and forth between different Hamiltonians
• Often uses different variables for different parts
• Then:

\[ \text{Error} = \epsilon (\Delta t)^{p+1} \left[H_0, H_{\text{pert}}\right] \]
Example: Leap-Frog

\[ H = \frac{1}{2} p^2 + \Phi(x) \]

Diagram:

1/2 Drift → Kick → 1/2 Drift

Cycle arrow above.
Example: SWIFT/MERCURY

\[ H = \frac{1}{2} p^2 + \Phi_{\text{Kepler}}(x) + \Phi_{\text{Other}}(x) \]
Example: Symplectic Epicycle Integrator

\[ H = \frac{1}{2} p^2 + \Omega (p \times r) e_z + \frac{1}{2} \Omega^2 \left[ r^2 - 3 (r \cdot e_x)^2 \right] + \Phi(r) \]
10 Orders of magnitude better!

non-symplectic

mixed variable, symplectic

symplectic

phase error

timestep $[2\pi\Omega^{-1}]$

Rein & Tremaine 2011
symplectic integrators = awesome
• Multi-purpose N-body code

• Optimized for collisional dynamics

• Code description paper recently accepted by A&A

• Written in C, open source

• Freely available at http://github.com/hannorein/rebound
REBOUND modules

**Gravity**
- Direct summation, $O(N^2)$
- BH-Tree code, $O(N \log(N))$
- FFT method, $O(N \log(N))$

**Collision detection**
- Direct nearest neighbor search, $O(N^2)$
- BH-Tree code, $O(N \log(N))$
- Plane sweep algorithm, $O(N)$ or $O(N^2)$

**Geometry**
- Open boundary conditions
- Periodic boundary conditions
- Shearing sheet / Hill's approximation

**Integrators**
- Leap frog
- Symplectic Epicycle integrator (SEI)
- Wisdom-Holman mapping (WH)
REBOUND scalings using a tree

**strong**

- MPI, 12.5k particles
- MPI, 50k particles
- MPI, 200k particles
- MPI, 800k particles

Linear scaling

**weak**

- MPI, 25k particles per node
- MPI, 50k particles per node
- MPI, 100k particles per node

1/\log(k)
DEMO
Take home message VII

Download REBOUND
Conclusions
Conclusions

Resonances and multi-planetary systems
Multi-planetary system provide insight in otherwise unobservable formation phase

- GJ876 formed in the presence of a disc and dissipative forces
- HD128311 formed in a turbulent disc
- HD45364 formed in a massive disc
- HD200964 did not form at all

Moonlets in Saturn’s rings
Small scale version of the proto-planetary disc
Random walk can be directly observed
Caused by collisions and gravitational wakes

REBOUND
N-body code, optimized for collisional dynamics, uses symplectic integrators
Open source, freely available, very modular and easy to use
http://github.com/hannorein/rebound