Multi-planetary systems, Saturn's Rings and the collisional N-body code REBOUND

Hanno Rein @ Rochester, November 2011
Planet formation
Migration in a non-turbulent disc
Gap opening criteria

\[
\frac{3}{4} \frac{H}{R_{\text{Hill}}} + \frac{50 M_*}{M_p \mathcal{R}} \leq 1
\]

Disc scale height  \rightarrow  \text{Stellar mass}  \rightarrow  \text{Planet mass}  \rightarrow  \text{Viscosity}
Migration - Type I

- Low mass planets
- No gap opening in disc
- Migration rate is fast
- Depends strongly on thermodynamics of the disc
Migration - Type II

- Massive planets (typically bigger than Saturn)
- Opens a (clear) gap
- Migration rate is slow
- Follows viscous evolution of the disc
Migration - Type III

• Massive disc
• Intermediate planet mass
• Ties to open gap
• Very fast, few orbital timescales
Resonance capture
2:1 Mean Motion Resonance

Star

Planet 1

Planet 2
2:1 Mean Motion Resonance
2:1 Mean Motion Resonance

\[ \frac{\lambda_2}{\lambda_1} = 2 \]

\[ \Delta \varpi \]
Resonant angles

- Fast varying angles
  \[ \lambda_1 - \omega_1 \quad \lambda_2 - \omega_2 \]

- Slowly varying combinations
  \[ \phi_1 = \lambda_2 - 2\lambda_1 + \omega_2 \]
  \[ \phi_2 = \lambda_2 - 2\lambda_1 + \omega_1 \]
  \[ \Delta \omega = \omega_1 - \omega_2 \]

- Two are linear independent
Non-turbulent resonance capture: two planets

\[ \phi_1 = \lambda_2 - 2\lambda_1 + \phi_2 \]
planet + disc = migration

2 planets + migration = resonance
HD 45364
HD45364

Observations vs Correia et al

Formation scenario for HD45364

- Two migrating planets
- Infinite number of resonances
- Migration speed is crucial
- Resonance width and libration period define critical migration rate

Fig. 1. Period ratio vs. migration timescale for different resonances. The inner planet is initially placed at 2R_e i, Papaloizou and Kley: The Dynamical Origin of HD 45364.
Formation scenario for HD45364

The semi-major axes, period ratio, and eccentricity are shown as functions of time. The semi-major axes decrease over time, the period ratio approaches a 3:2 commensurability, and the eccentricity increases initially before settling into a steady state. The plots are color-coded to show the evolution of the system over time. The top panel shows the semi-major axes, the middle panel the period ratio, and the bottom panel the eccentricity. The diagrams on the right show the evolution of the system at different time points, illustrating the migration and dynamics of the planets.
Formation scenario for HD45364

Massive disc (5 times MMSN)

- Short, rapid Type III migration
- Passage of 2:1 resonance
- Capture into 3:2 resonance

Large scale-height (0.07)

- Slow Type I migration once in resonance
- Resonance is stable
- Consistent with radiation hydrodynamics

Rein, Papaloizou & Kley 2010
Formation scenario leads to a better ‘fit’

![Graph showing radial velocity vs. JD-2400000 [days].]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Correia et al. (2009)</th>
<th>Simulation F5</th>
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<tbody>
<tr>
<td></td>
<td>b</td>
<td>c</td>
<td></td>
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<tr>
<td>$M \sin i$</td>
<td>[M$_{\text{Jup}}$]</td>
<td>0.1872</td>
<td>0.1872</td>
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<tr>
<td>$M_*$</td>
<td>[M$_{\odot}$]</td>
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<td>0.6579</td>
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<td>$a$</td>
<td>[AU]</td>
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<td>$e$</td>
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<td>0.6813</td>
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<tr>
<td></td>
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<td>0.097 ± 0.012</td>
<td>0.8994</td>
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<tr>
<td>$\lambda$</td>
<td>[deg]</td>
<td>105.8 ± 1.4</td>
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<td></td>
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<td>269.5 ± 0.6</td>
<td>153.9</td>
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<tr>
<td>$\varpi_a$</td>
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<td>7.4 ± 4.3</td>
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<td>$\sqrt{\chi^2}$</td>
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<td>2.79</td>
<td>2.76$^b$ (3.51)</td>
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<tr>
<td>Date</td>
<td>[JD]</td>
<td>2453500</td>
<td>2453500</td>
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Rein, Papaloizou & Kley 2010
Migration in a turbulent disc
Turbulent disc

- Angular momentum transport
- Magnetorotational instability (MRI)
- Density perturbations interact gravitationally with planets
- Stochastic forces lead to random walk
- Large uncertainties in strength of forces

Animation from Nelson & Papaloizou 2004
Random walk

- pericenter
- eccentricity
- semi-major axis

(time)
Correction factors are important

\[ (\Delta a)^2 = 4 \frac{Dt}{n^2} \]

\[ (\Delta \omega)^2 = \frac{2.5 \gamma Dt}{e^2 n^2 a^2} \]

\[ (\Delta e)^2 = 2.5 \frac{\gamma Dt}{n^2 a^2} \]

Two planets: turbulent resonance capture

Rein & Papaloizou 2009
Multi-planetary systems in mean motion resonance

- Stability of multi-planetary systems depends strongly on diffusion coefficient
- Most planetary systems are stable for entire disc lifetime

Rein & Papaloizou 2009
Modification of libration patterns

• HD128311 has a very peculiar libration pattern
• Can not be reproduced by convergent migration alone
• Turbulence can explain it
• More multi-planetary systems needed for statistical argument
HD200964
The impossible system?
Radial velocity curve of HD200964

- Two massive planets
  1.8 $M_{\text{Jup}}$ and 0.9 $M_{\text{Jup}}$

- Period ratio either
  3:2 or 4:3

- Another similar system, to be announced soon

- How common is 4:3?

- Formation?

Plot by Matthew Payne
N-body simulations

The diagram shows the relationship between migration timescale and mass ratio. The period ratio is represented on the right side of the graph. The x-axis represents the mass ratio, while the y-axis represents the migration timescale. The color gradient indicates the period ratio values.
Hydrodynamical simulations
Stability of HD200964
• In situ formation?

• Main accretion while in 4:3 resonance?

• Planet planet scattering?

• A third planet?

• Observers screwed up?
dynamical state of planetary systems

formation scenario
Moonlets in Saturn's Rings
Propeller structures in A-ring

Porco et al. 2007, Sremcevic et al. 2007, Tiscareno et al. 2006
Longitude residual

Mean motion [rad/s]
\[ n = \sqrt{\frac{GM}{a^3}} \]

Mean longitude [rad]
\[ \lambda = n t \]

\[ \lambda(t) - \lambda_0(t) = \int_0^t (n_0 + n'(t')) \, dt' - \int_0^t n_0 \, dt' - n_0 t \]
Observational evidence of non-Keplerian motion

Figure 4. Observed longitude of the propeller "Blériot" over 4 years with a linear trend. Only data points with measurement errors $\sigma < k$ are shown. Error bars are given but in many cases are smaller than the plotting symbol. Panel c shows all the data while panels cd, cc, and cd contain subsets of the data shown in greater detail. The residuals to the linear trend (horizontal dotted line) are less than $\pm k$ but are clearly not randomly distributed. The dotted line indicates a linear plus sinusoidal fit to all the data with an amplitude of $k$. Panel cd shows all the data, while panels cc and cd contain subsets of the data shown in greater detail. The residuals to the linear trend (horizontal dotted line) are less than $\pm k$ but are clearly not randomly distributed. The dotted line indicates a linear plus sinusoidal fit to all the data with an amplitude of $k$.

Table 1

<table>
<thead>
<tr>
<th>Nickname</th>
<th>Longitude Rms deviation</th>
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<tbody>
<tr>
<td>Earhart</td>
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<td>Blériot</td>
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Formal error estimates shown in parentheses for the last digit are for the best-fit linear trend in longitude. They are much smaller than the rms deviations in longitude given in the right-hand column.

Epoch is January at UTC. All orbit fits assume $e$ and $i$.

Not including images of insufficient quality to include in the orbit fits exclusively proven that giant propellers are missing in the Propeller Belts. Even the largest propellers observed in the Propeller Belts have $\Delta r < w$ km. Tiscareno et al. 2010

The orbital evolution of "Blériot"

At least $w$ propellers have been seen at multiple widely separated instances, but "Blériot" is of particular interest as the largest and most frequently detected. It has appeared in more than one hundred separate Cassini ISS images spanning a period of four years, and was serendipitously detected once in a stellar occultation observed by the Cassini UVIS instrument. Analysis of the orbit of "Blériot" confirms that it is both long-lived and reasonably well-characterized by a keplerian path. As Figure 4 shows, a linear fit to the longitude with time (corresponding to a circular orbit) results in residuals of $\pm k$ longitude.
Random walk

**Analytic model**
Describing evolution in a statistical manner
Partly based on Rein & Papaloizou 2009

\[
\Delta a = \sqrt{\frac{4Dt}{n^2}} \\
\Delta e = \sqrt{\frac{2.5\gamma Dt}{n^2a^2}}
\]

**N-body simulations**
Measuring random forces or integrating moonlet directly
Crida et al 2010, Rein & Papaloizou 2010

Rein & Papaloizou 2010, Crida et al 2010
Random walk

REBOUND code, Rein & Papaloizou 2010, Crida et al 2010
Figure 4: Observed longitude of the propeller "Blériot" over 4 years, with a linear trend after subtracting only data points with measurement errors \( \sigma < k \). Error bars are given but in many cases are smaller than the plotting symbols. Panel a shows all the data, while panels b-d contain subsets of the data shown in greater detail. The residuals to the linear trend (horizontal dotted line) are less than \( \pm k \) but are clearly not randomly distributed. The dotted line indicates a linear plus sinusoidal fit to all the data with an amplitude of \( k \) and a period of \( n \). The solid lines indicate piecewise quadratic fits corresponding to a constant drift in semimajor axis in particular, the data from mid- to early- are fit by a linear trend with a constant acceleration of \( k \). while the data from late- to early- are fit by a linear trend with a constant acceleration of \( k \).

Table 1: Orbit fits for trans-Encke propellers

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Not including images of insufficient quality to include in the orbit fitting. That giant propellers are missing in the Propeller Belts, even the largest propellers observed in the Propeller Belts have \( \Delta r < w \). while nearly all observed trans-Encke propellers have \( \Delta r \) larger than this value. Figure 3.

The orbital evolution of "Blériot" at least 4 propellers have been seen at multiple widely separated instances, but "Blériot" is of particular interest as the largest and most frequently detected. It has appeared in more than one hundred separate Cassini ISS images spanning a period of four years, and was serendipitously detected once in a stellar occultation observed by the Cassini UVIS instrument. Analysis of the orbit of "Blériot" confirms that it is both long-lived and reasonably well-characterized by a keplerian path. As Figure 4 shows, a linear fit to the longitude with time (corresponding to a circular orbit) results in residuals of \( \pm k \) km in longitude. Hows...
Saturn's rings
=
small scale version of
a proto-planetary disc
REBOUND
A new open source collisional N-body code
Numerical Integrators

• We want to integrate the equations of motions of a particle

\[ \dot{x} = v \]
\[ \dot{v} = a(x, v) \]

• For example, gravitational potential

\[ a(x) = -\nabla \Phi(x) \]

• In physics, these can usually be derived from a Hamiltonian

\[ H = \frac{1}{2}p^2 + \Phi(x) \]

• Symmetries of the Hamiltonian correspond to conserved quantities
Numerical Integrators

• Discretization
  \[ \dot{x} = v \quad \Rightarrow \quad \Delta x = v \Delta t \]
  \[ \dot{v} = a(x, v) \quad \Rightarrow \quad \Delta v = a(x, v) \Delta t \]

• Hamiltonian
  \[ H = \frac{1}{2} p^2 + \Phi(x) \]

• The system is governed by a 'discretized Hamiltonian', if and only if the integration scheme is symplectic.

• Why does it matter?
Symplectic vs non symplectic integrators
Mixed variable integrators

• So far: symplectic integrators are great.
• How can it be even better?
• We can split the Hamiltonian:

\[ H = H_0 + \epsilon H_{\text{pert}} \]

Integrate particle exactly with dominant Hamiltonian

Integrate particle exactly under perturbation Hamiltonian

• Switch back and forth between different Hamiltonians
• Often uses different variables for different parts
• Then:

\[ \text{Error} = \epsilon (\Delta t)^{p+1} \left[ H_0, H_{\text{pert}} \right] \]
Example: Leap-Frog

\[ H = \frac{1}{2} p^2 + \Phi(x) \]
Example: SWIFT/MERCURY

\[ H = \frac{1}{2} p^2 + \Phi_{\text{Kepler}}(x) + \Phi_{\text{Other}}(x) \]

1/2 Kick \rightarrow Kepler \rightarrow Kick \rightarrow 1/2 Kick
Example: Symplectic Epicycle Integrator

\[ H = \frac{1}{2} p^2 + \Omega (p \times r) e_z + \frac{1}{2} \Omega^2 \left[ r^2 - 3(r \cdot e_x)^2 \right] \]

Epicycle

Kick

\[ \Phi(r) \]

1/2 Kick \rightarrow Epicycle \rightarrow 1/2 Kick
10 Orders of magnitude better!

non-symplectic

mixed variable, symplectic

symplectic

Rein & Tremaine 2011
symplectic integrators = awesome
- Multi-purpose N-body code
- Optimized for collisional dynamics
- Code description paper recently accepted by A&A
- Written in C, open source
- Freely available at http://github.com/hannorein/rebound
**Geometry**
- Open boundary conditions
- Periodic boundary conditions
- Shearing sheet / Hill's approximation

**Gravity**
- Direct summation, $O(N^2)$
- BH-Tree code, $O(N \log(N))$
- FFT method, $O(N \log(N))$

**Integrators**
- Leap frog
- Symplectic Epicycle integrator (SEI)
- Wisdom-Holman mapping (WH)

**Collision detection**
- Direct nearest neighbor search, $O(N^2)$
- BH-Tree code, $O(N \log(N))$
- Plane sweep algorithm, $O(N)$ or $O(N^2)$

**Geometry**
- Open boundary conditions
- Periodic boundary conditions
- Shearing sheet / Hill's approximation

**Gravity**
- Direct summation, $O(N^2)$
- BH-Tree code, $O(N \log(N))$
- FFT method, $O(N \log(N))$
REBOUND scalings using a tree

**strong**

- MPI, 12.5k particles
- MPI, 50k particles
- MPI, 200k particles
- MPI, 800k particles
- linear scaling

**weak**

- MPI, 25k particles per node
- MPI, 50k particles per node
- MPI, 100k particles per node
- $1/\log(k)$
Download REBOUND
Conclusions
Conclusions

**Resonances and multi-planetary systems**
Multi-planetary system provide insight in otherwise unobservable formation phase

- GJ876 formed in the presence of a disc and dissipative forces
- HD128311 formed in a turbulent disc
- HD45364 formed in a massive disc
- HD200964 did not form at all

**Moonlets in Saturn’s rings**
Small scale version of the proto-planetary disc
Random walk can be directly observed
Caused by collisions and gravitational wakes

**REBOUND**
N-body code, optimized for collisional dynamics, uses symplectic integrators
Open source, freely available, very modular and easy to use
http://github.com/hannorein/rebound