Simulating planetary systems

Newton's law of gravity, machine learning,
and everything in-between

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Image credit: NASA/SOFIA/Lynette Cook

Agenda

- **Early work on Solar System Dynamics**
- **Chaos**
- **Modern numerical methods**
- **Machine learning**
- **REBOUND, REBOUNDx, ASSIST**
- **Light pollution**

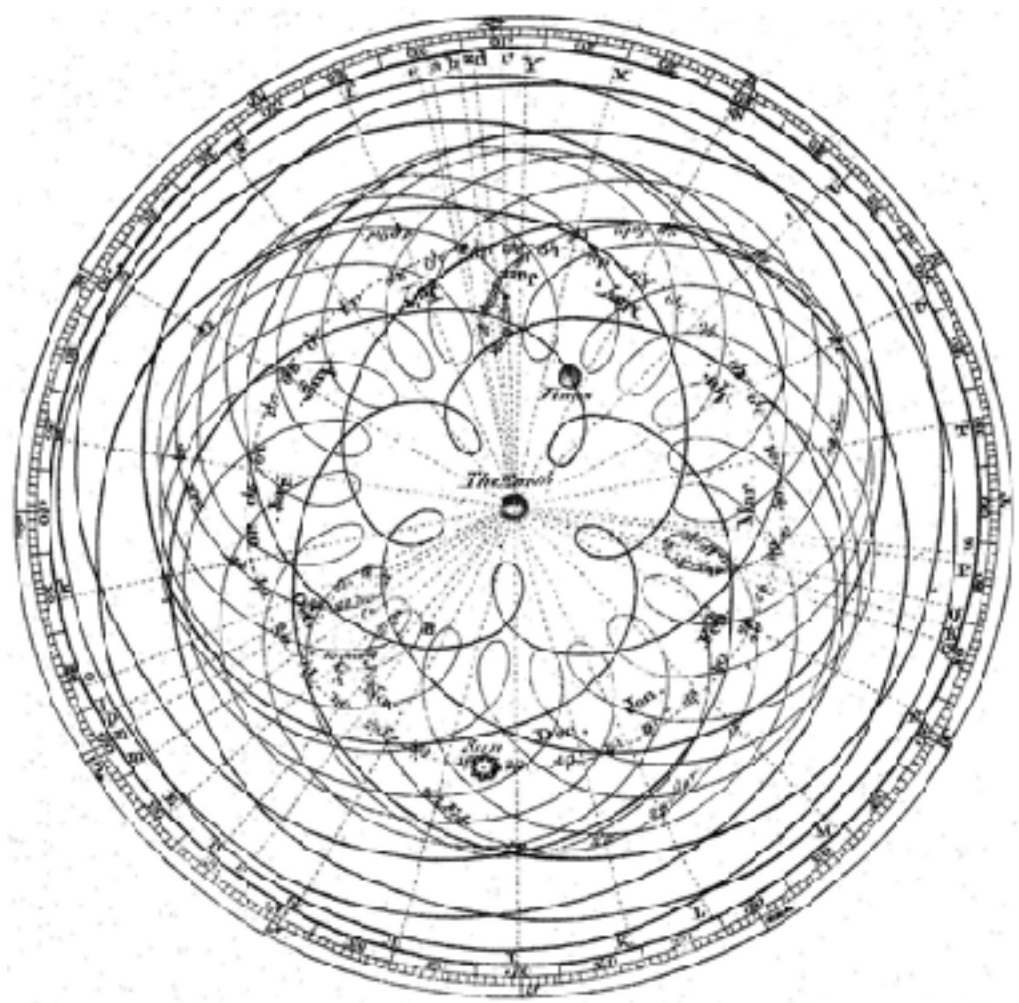
Early work on Solar System Dynamics

Laskar (*Lagrange et la stabilité du Systéme solaire*, 2006), Laskar (2013)

Newton (1687)

$$\ddot{\mathbf{r}}_i = \sum_{\substack{j=1 \\ j \neq i}}^{N} m_j \frac{\mathbf{r}_j - \mathbf{r}_i}{\left|\mathbf{r}_j - \mathbf{r}_i\right|^3}$$

Newton (Opticks 1717,1730)



For while comets move in very excentrick orbs in all manner of positions, blind fate could never make all the planets move one and the same way in orbs concentrick, some inconsiderable irregularities excepted, which may have risen from the **mutual actions** of comets and planets upon one another, and which will be apt to increase, **till this system wants a reformation.**

Evidence for irregularity/instability

Ptolemy

On March 1st, 228 BC, at 4:23 am, mean Paris time, Saturn was observed two fingers under Gamma in Virgo.

+

Observations from 1590 and 1650.

=

Six million years ago Jupiter and Saturn were at the same distance from the Sun.

Demo with REBOUND

Laplace-Lagrange Secular Dynamics

Average over short time scales

Perturbation theory

Explanations for the irregularities?

Euler was twice awarded a prize in 1748 and 1752 related to this problem by the Paris Academy of Sciences.

Lagrange thought that Euler’s calculations were wrong and did his own.

Laplace (1776)

Mr. Euler, in his second piece on the irregularities of Jupiter and Saturn, find it **equal for both these planets**. According to Mr. de Lagrange, on the contrary, […] it is very **different for these two bodies**. […] I have some reasons to believe, however, that the formula is still not accurate. The one which I obtain is quite different. […] by substituting these values in the formula of the secular equation, **I found absolutely zero**, from which I conclude the alteration of the mean motion of Jupiter, if it exists, does not result from the action of Saturn.

Lagrange (in a letter to d'Alembert, 1775)

I am ready to give a **complete theory for the variations of the elements of the planets** under their mutual action. That Mr. de la Place did on this subject I liked, and I flatter myself that he will not be offended if I do not hold the kind of promise that I made to completely abandon this subject to him; I could not resist to the desire to look into it again, but I am no less charmed that he is also working on it on his side; I am even very eager to read his subsequent research on this topic, but I do ask him not to send me any manuscript and send them to me only in printed form.

Fundamental modes, eigenfrequencies

	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8
Lagrange (1774)	5.98	6.31	19.80	18.31	0	25.34	-	-
Brown & Rein (2019)	5.59	7.05	18.84	17.74	0	26.35	2.99	0.69

Note: No semi-major axis changes to first and also second order (Poisson, Haretu and Poincaré) in the expansion.

This still contradicts Ptolemy's observations from antiquity.

Demo with REBOUND

Laplace (1785)

Simple energy argument implies:

$$\frac{m_J}{a_J} + \frac{m_S}{a_S} = const$$

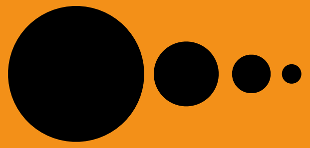
Thus, can be confident that the change in orbits must be due to mutual interactions.

He's also shown, no secular terms. Hence must be short period.

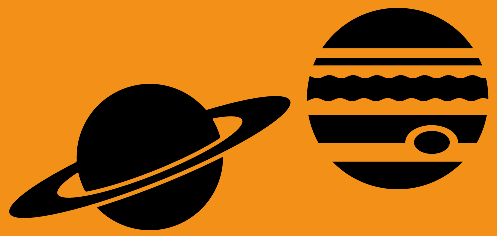
Near 5:2 mean motion resonance. Period of 900 years.

Secular Dynamics

Average over
short time scales



Perturbation theory



5:2 near MMR

Secular Dynamics

Average over
short time scales

ory

5:2 near MMR

Accurate over
> 1 million years

Chaos

Le Verrier (1840, 1841)

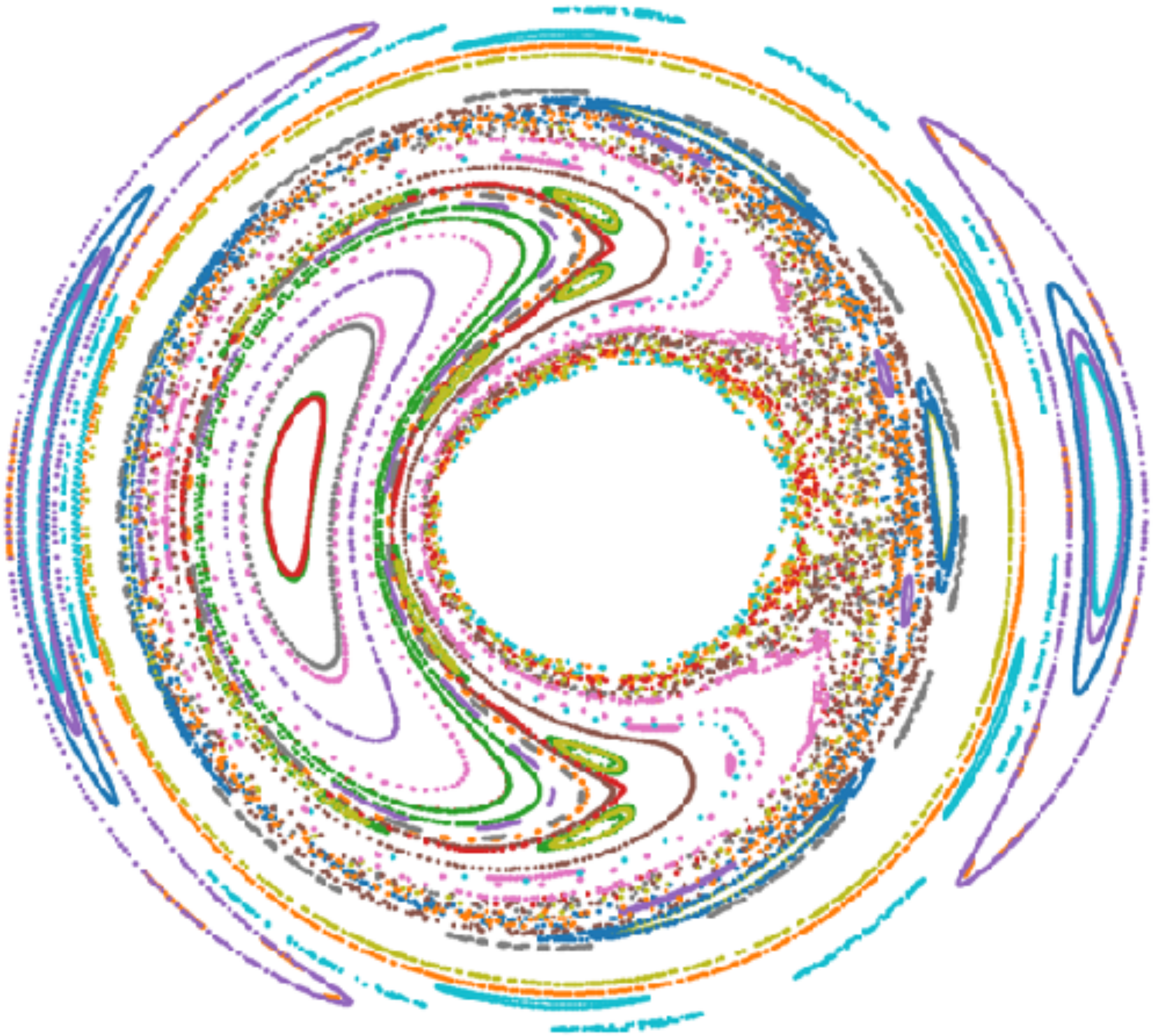
Follow up on the work of Lagrange and Laplace but to higher order.

Small divisor problem:
third order could be larger than second order terms

Poincaré (1897)

The terms of these series, in fact, decrease first very quickly and then begin to grow, **but as the Astronomers' stop after the first terms of the series,** and well before these terms have stop to decrease, the approximation is sufficient for the practical use. The divergence of these expansions would have some disadvantages only if one wanted to use them to rigorously establish some specific results, as the **stability of the Solar System**.

Kolmogorov (1954), Arnold (1963), Moser (1962)

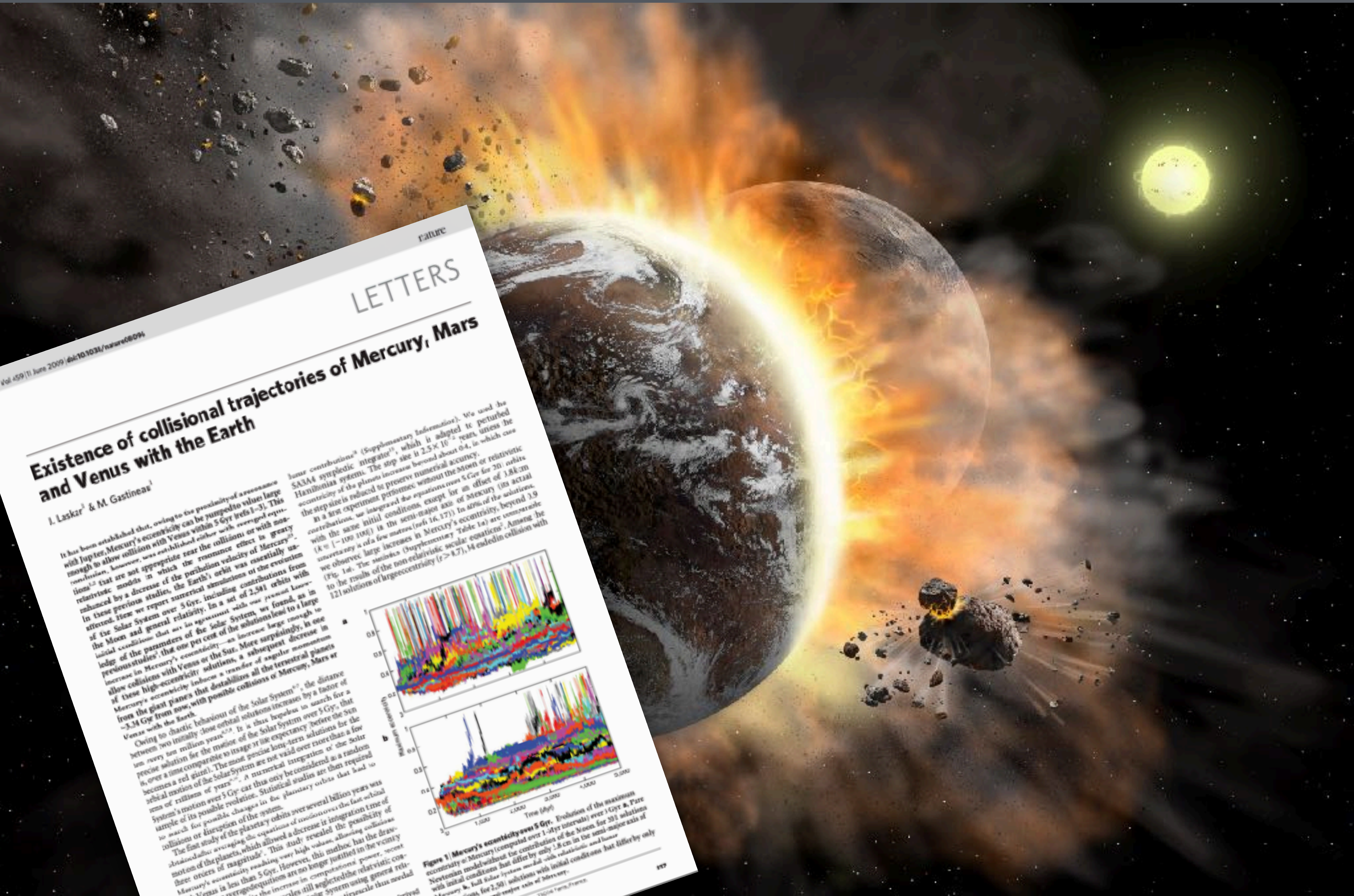


Kolmogorov showed that convergent perturbation series can exist.

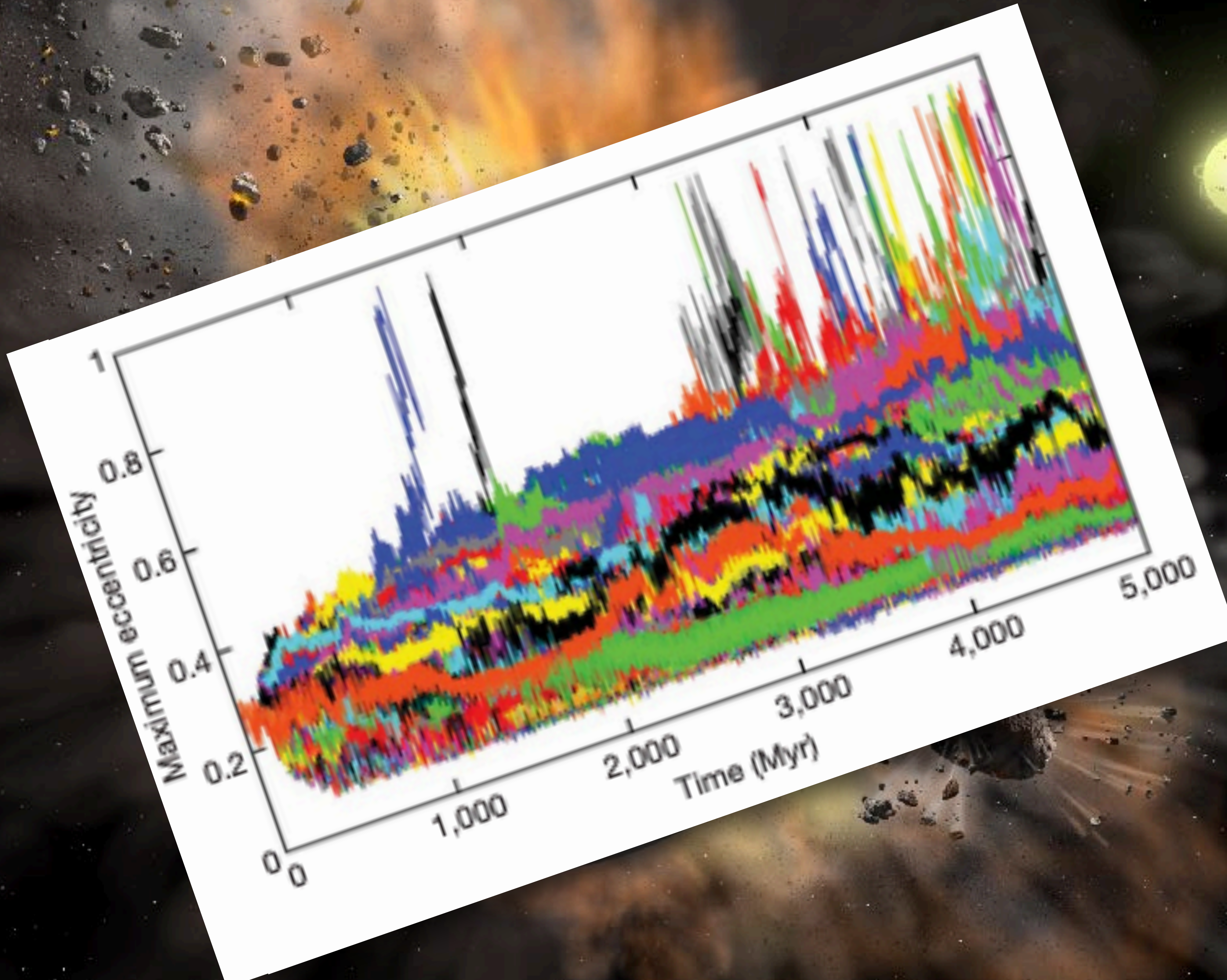
Many subtleties (degeneracy, small masses, slow Arnold diffusion)

In short: expansions are not useful for determining the stability of our Solar System.

Laskar (2009)


nature
LETTERS
Existence of collisional trajectories of Mercury, Mars and Venus with the Earth
J. Laskar & M. Gastineau

Laskar (2009)



Maximum eccentricity
Time (Myr)
0
0.2
0.4
0.6
0.8
1
0
1,000
2,000
3,000
4,000
5,000

Modern numerical methods

Newton (1687)

$$\ddot{\mathbf{r}}_i = \sum_{\substack{j=1 \\ j \neq i}}^{N} m_j \frac{\mathbf{r}_j - \mathbf{r}_i}{\left|\mathbf{r}_j - \mathbf{r}_i\right|^3}$$

How to solve N-body ODE

Brute force approach
(IAS15)

Physical approach
(WHFast/EOS)

Fundamentals

Equations of motions

$$\ddot{\mathbf{r}}_i = \sum_{\substack{j=1 \\ j\neq i}}^{N} m_j \frac{\mathbf{r}_j - \mathbf{r}_i}{|\mathbf{r}_j - \mathbf{r}_i|^3}$$

↕

kinetic term

potential term

Hamiltonian

$$H = T + U$$

Leap frog integrator

$$H = T + U$$

A $$T = \sum_{i=0}^{N-1} T_i$$

B $$U = \sum_{i=0}^{N-1} \sum_{j=i+1}^{N-1} U_{ij}$$

Both solutions are trivial!

$$\dot{r}_i = v_i$$

$$\dot{v}_i = \sum_{j \neq i} a_{ij}$$

Wisdom-Holman integrator

$$H = A + B$$

A $$\sum_{i=0}^{N-1} T_i + \sum_{i=1}^{N-1} U_{i0}$$

Dominant part of motion

B $$\sum_{i=1}^{N-1} \sum_{j=i+1}^{N-1} U_{ij}$$

Perturbation

Solution for B is still trivial.

Solution for A is more complicated. We need a “Kepler solver”.

Rein & Tamayo (2015)

Embedded Operator Splitting Method (EOS)

$$H = A + B$$

A $\sum_{i=0}^{N-1} T_i + \sum_{i=1}^{N-1} U_{i0}$

B $\sum_{i=1}^{N-1} \sum_{j=i+1}^{N-1} U_{ij}$

$$A = A_1 + A_2$$

A_1 $\sum_{i=0}^{N-1} T_i$

A_2 $\sum_{i=1}^{N-1} U_{i0}$

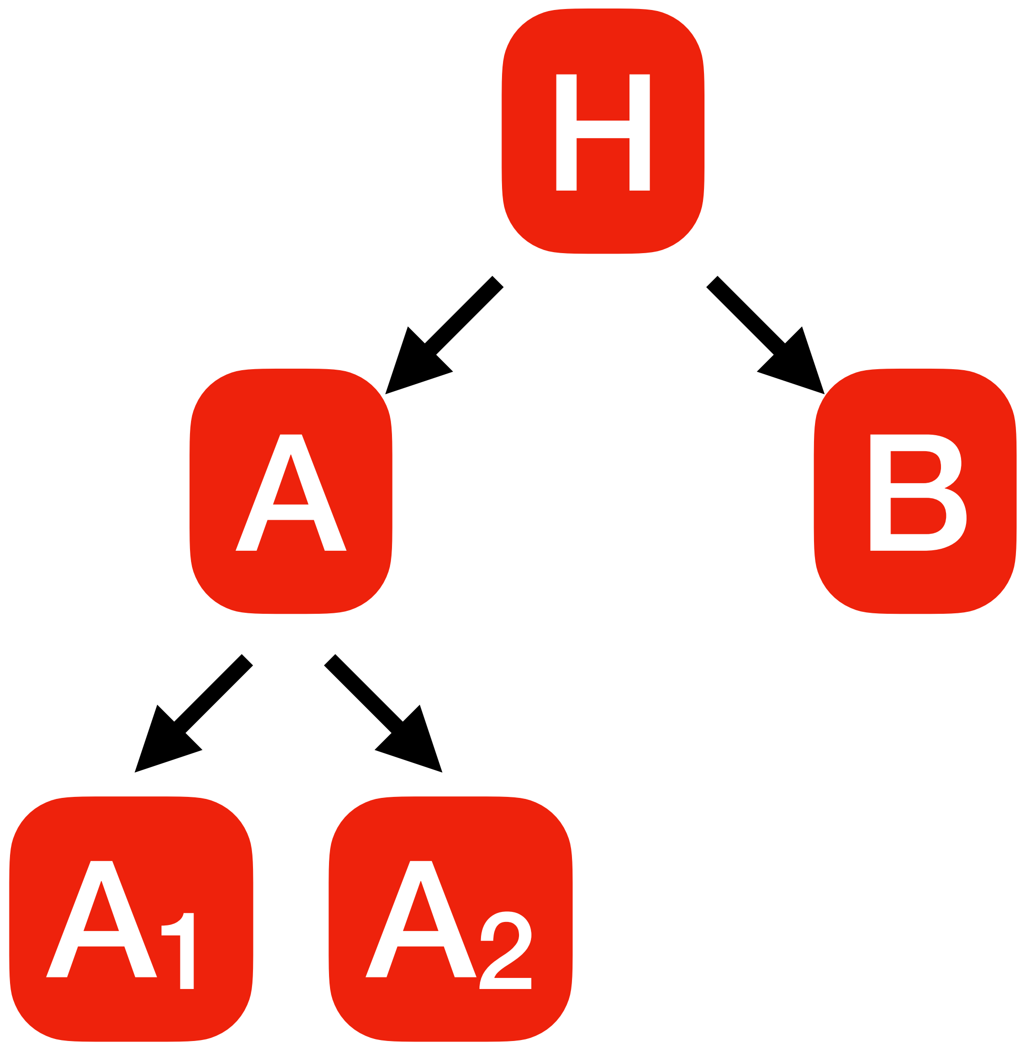
Rein (2020)

A lot of choice

Full Hamiltonian

1st splitting

2nd splitting



At each splitting, can choose:

- How to split Hamiltonian into two parts
- Which splitting method to use
- Timestep

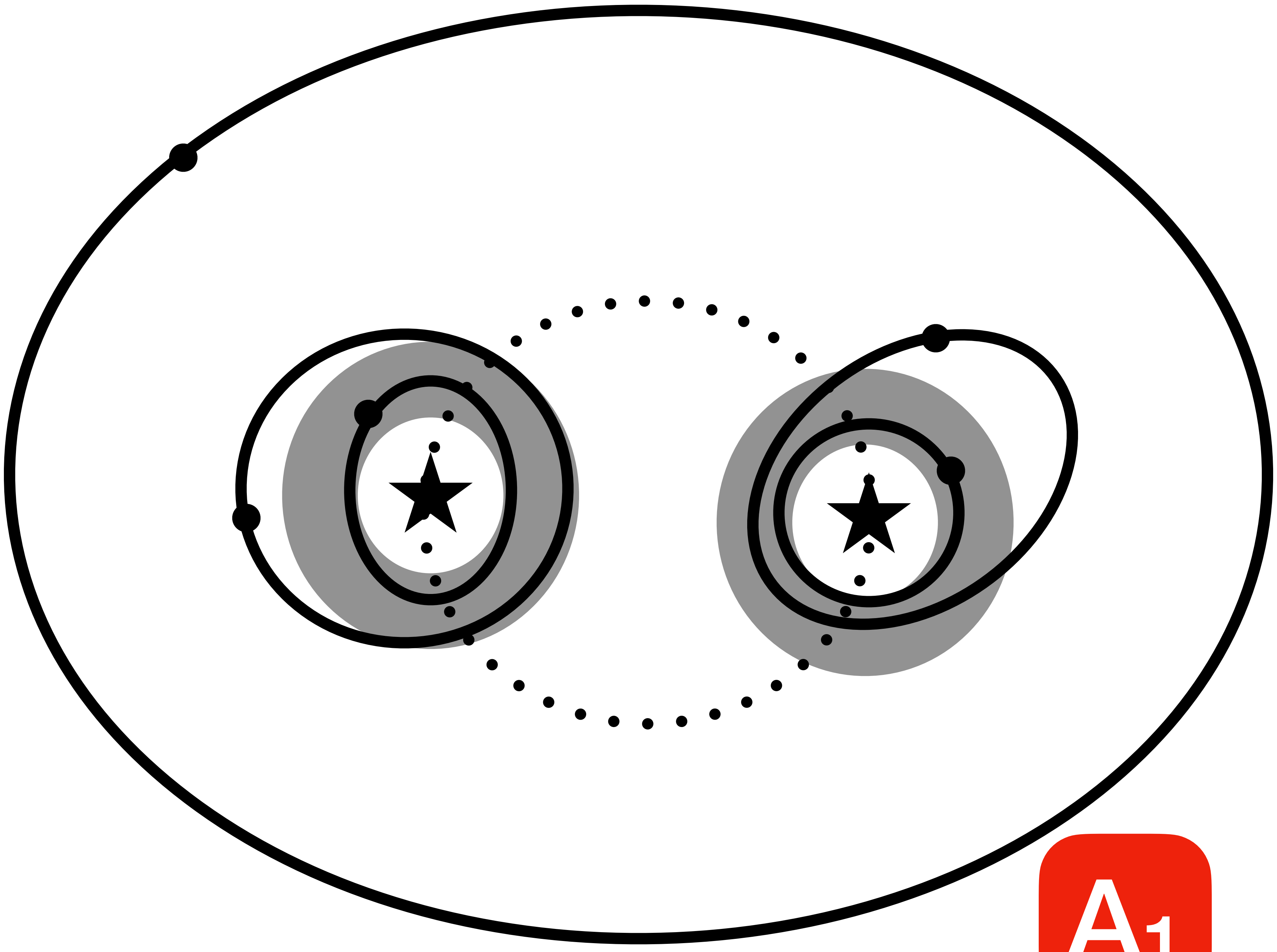
Rein (2020)

EOS methods are extremely flexible

Splitting	Method
splitting into T + U	leap frog, higher order leap frog
splitting into Keplerian motion + perturbations	Wisdom Holman integrator, higher order generalizations
splitting into near and far interactions	Hybrid symplectic integrators, Mercury
splitting into many different “shells”	SYMBA

Rein (2020)

Example: complicated hierarchical systems



A_1 kinetic term (drift)

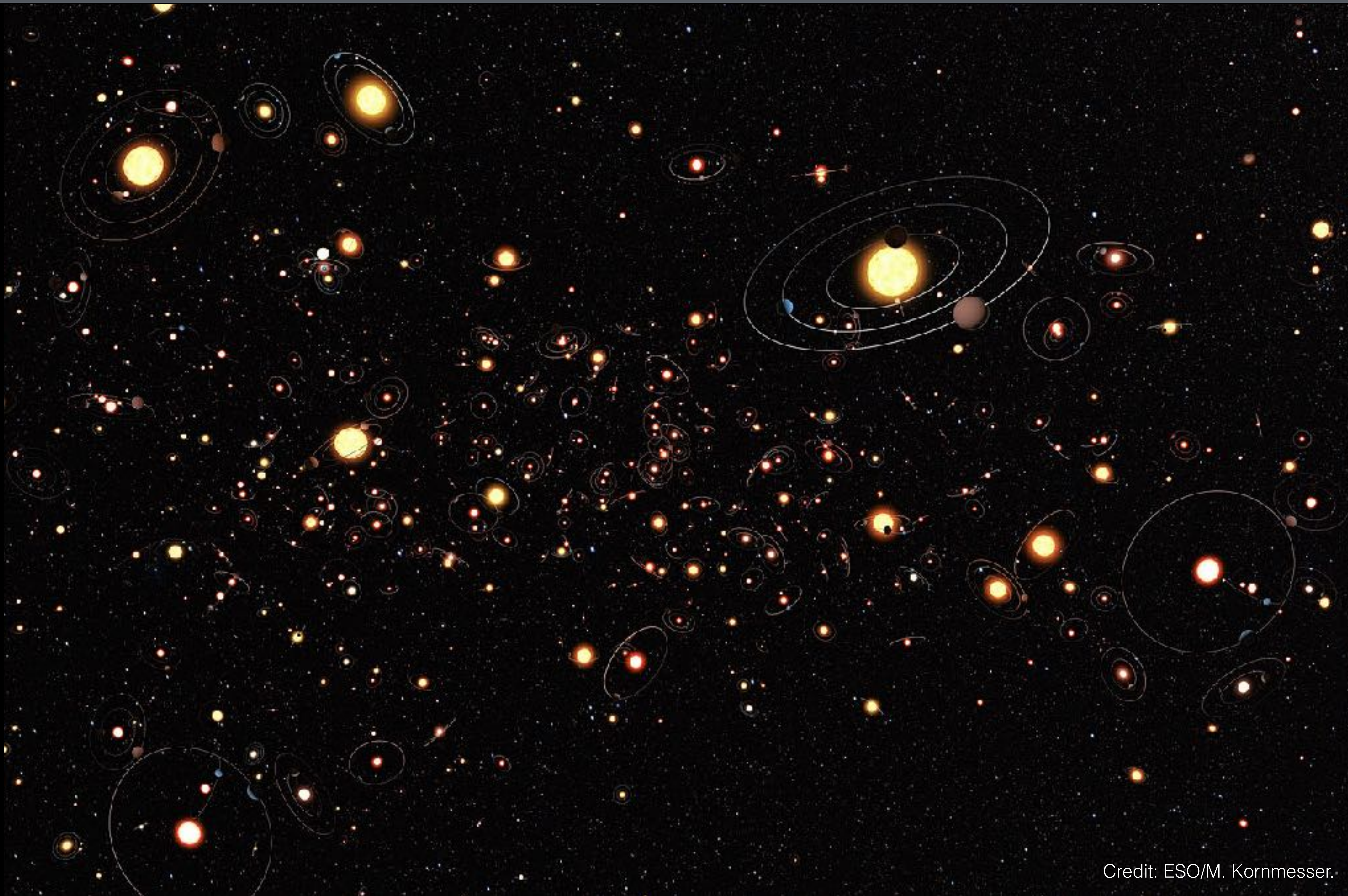
B planet-planet interactions (kick)

A_2 planet-star and star-star interactions (kick)

Predicting the stability of planetary systems with machine learning

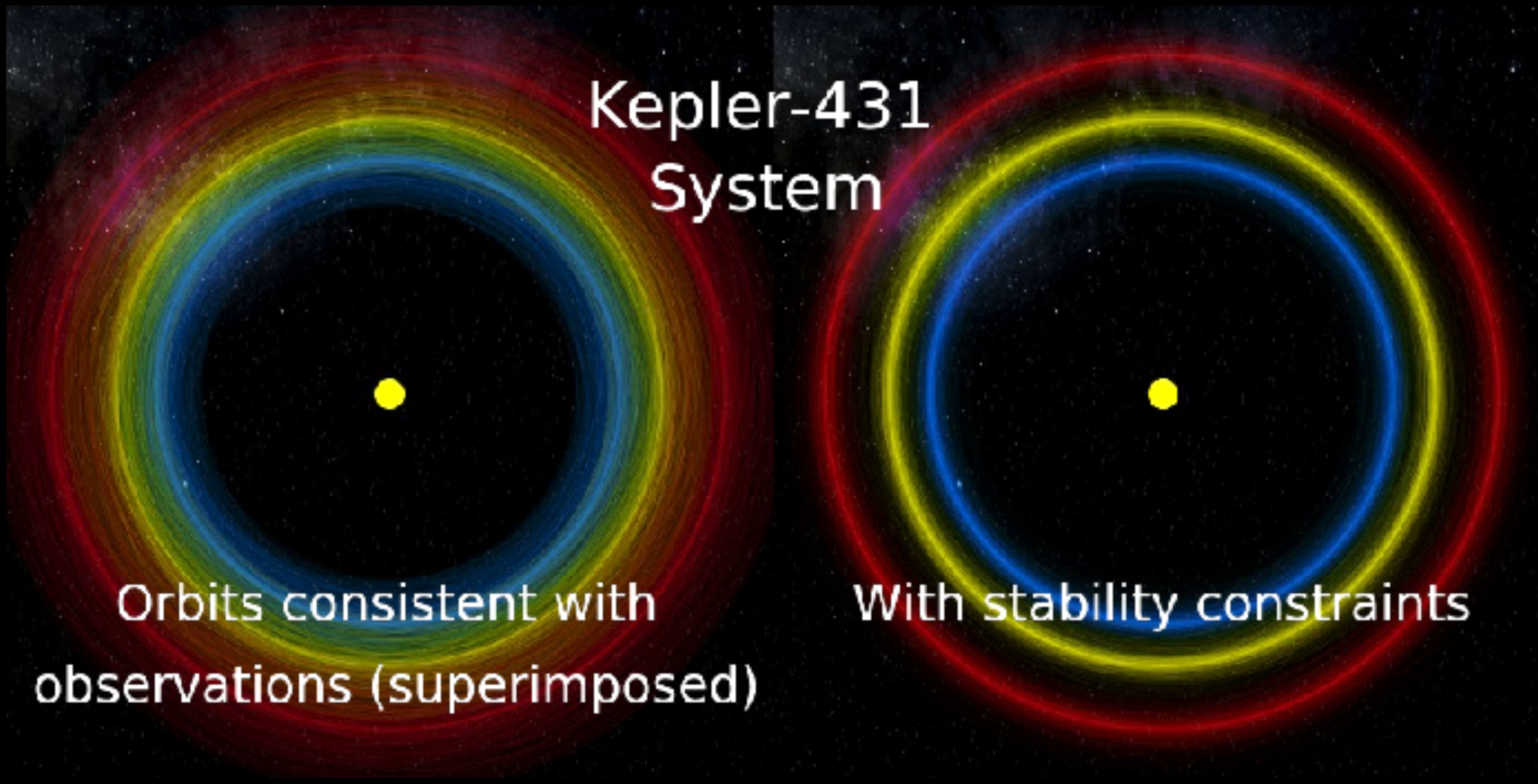
Tamayo et al. (submitted)

Planets everywhere



Credit: ESO/M. Kornmesser.

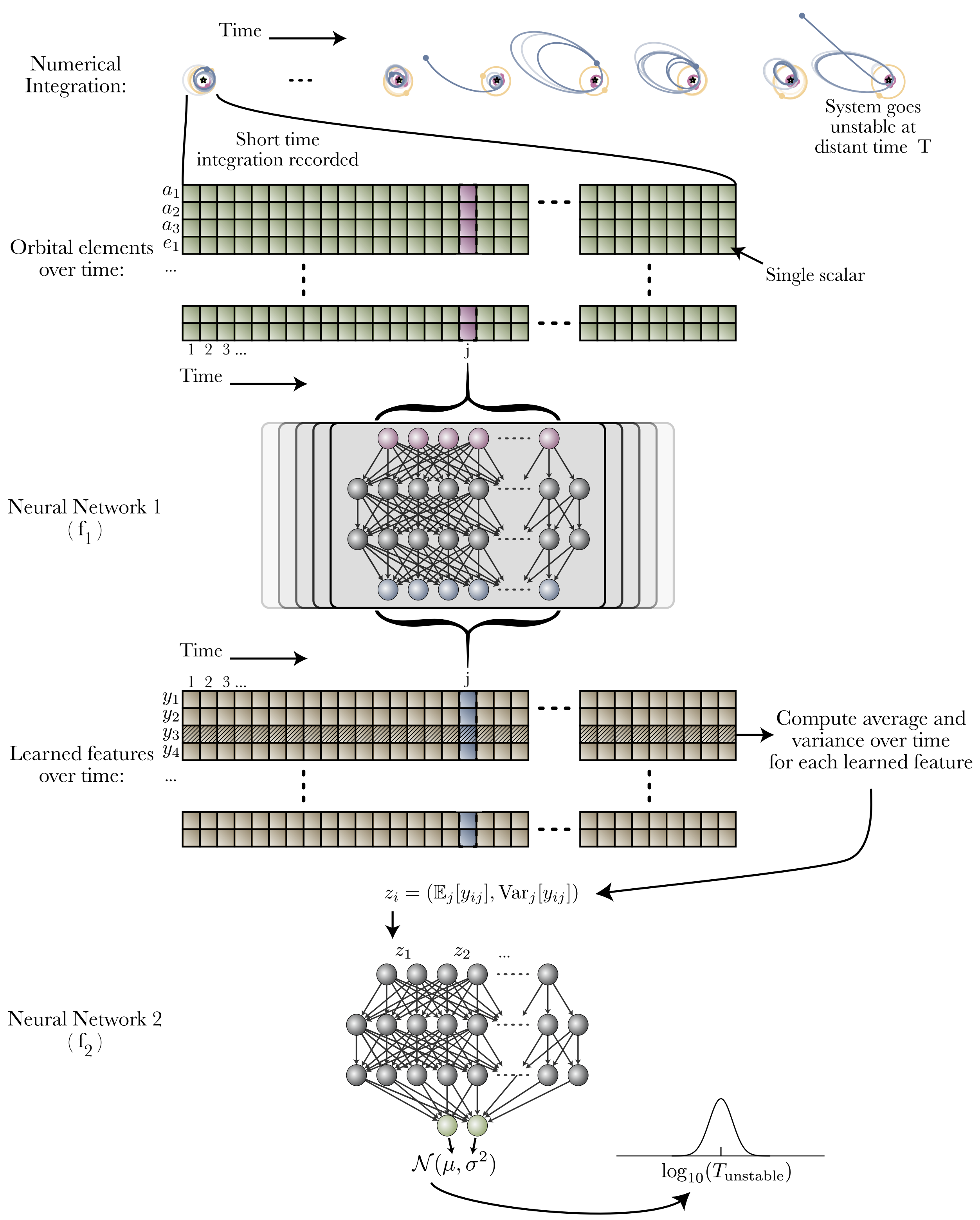
Constrain orbital parameters



- This is a hard problem!
- Machine learning models can help solve this problem.
- Doesn't have to be a black box! Can be part of a Bayesian analysis.

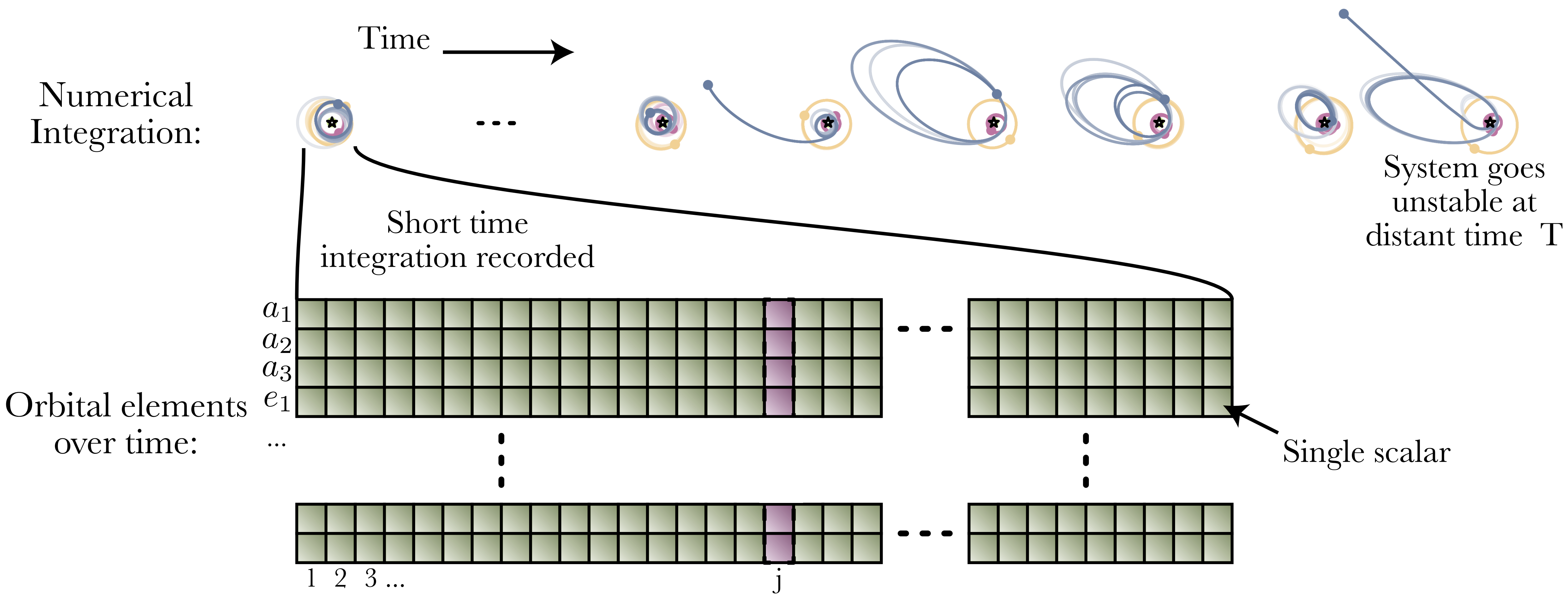
Tamayo et al. (2021)

Bayesian Neural Network



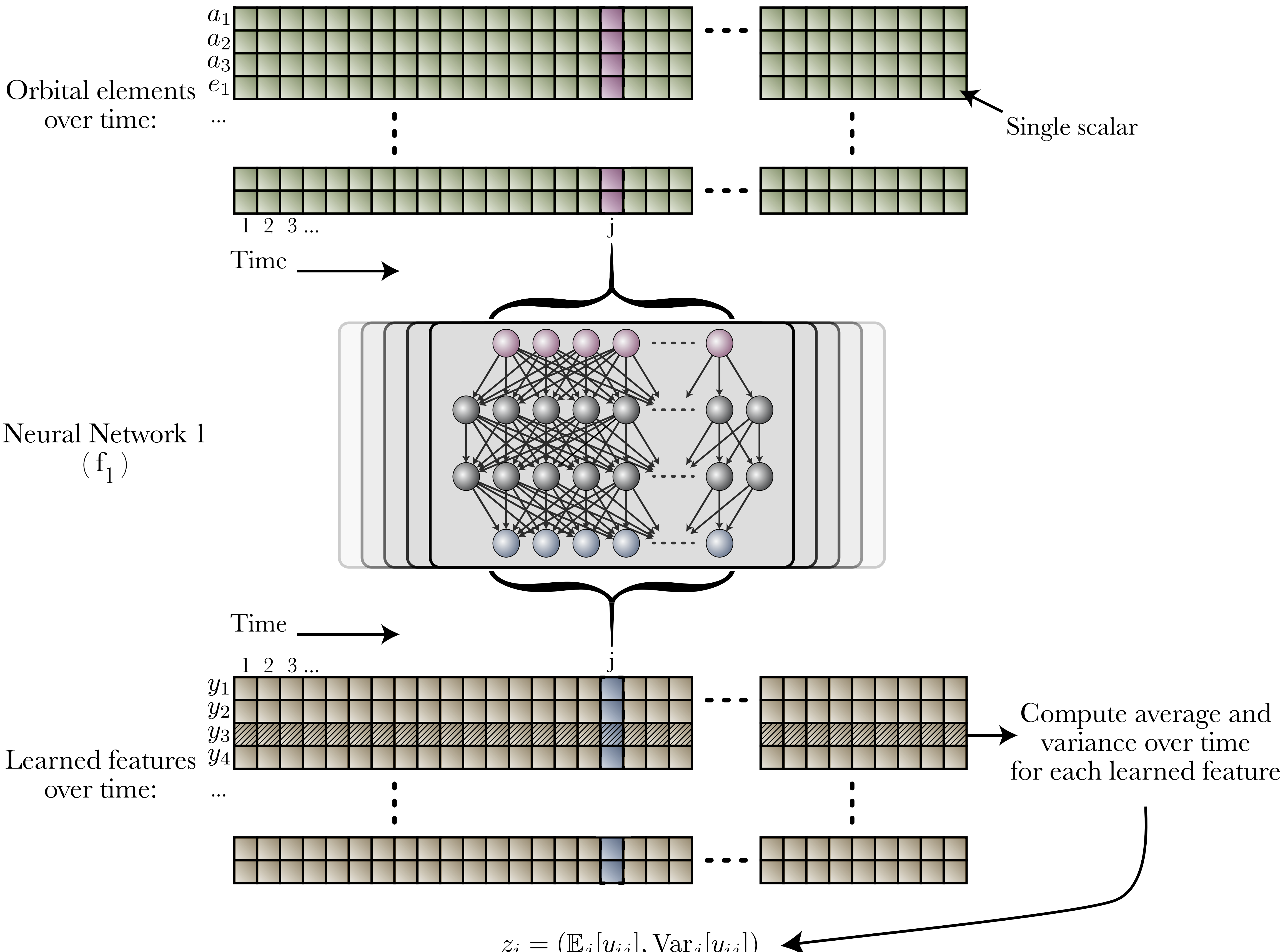
Cranmer et al (2021), Tamayo et al. (2020)

Step 1: Short integration



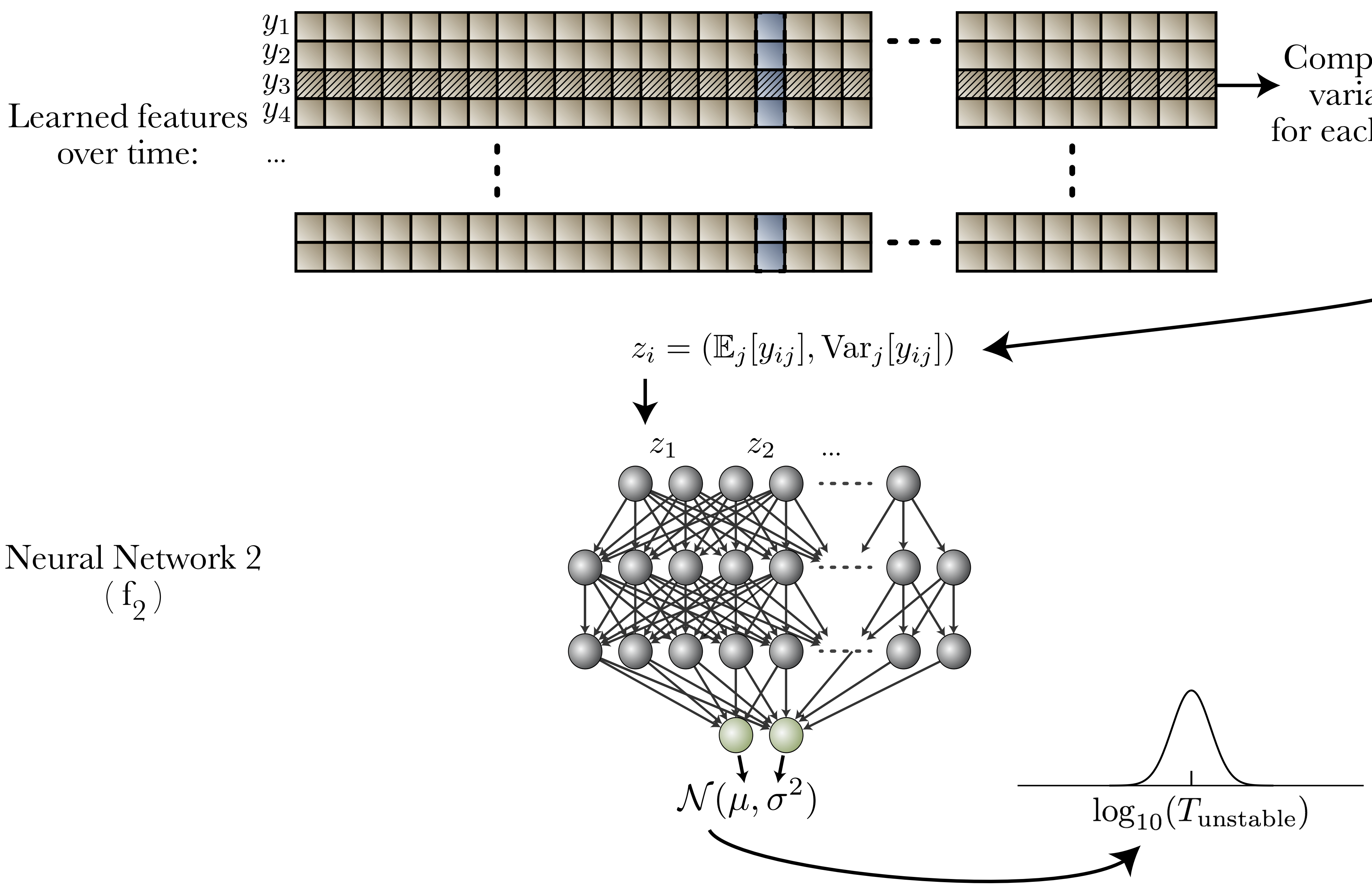
Cranmer et al (2021), Tamayo et al. (2020)

Step 2: Learning features



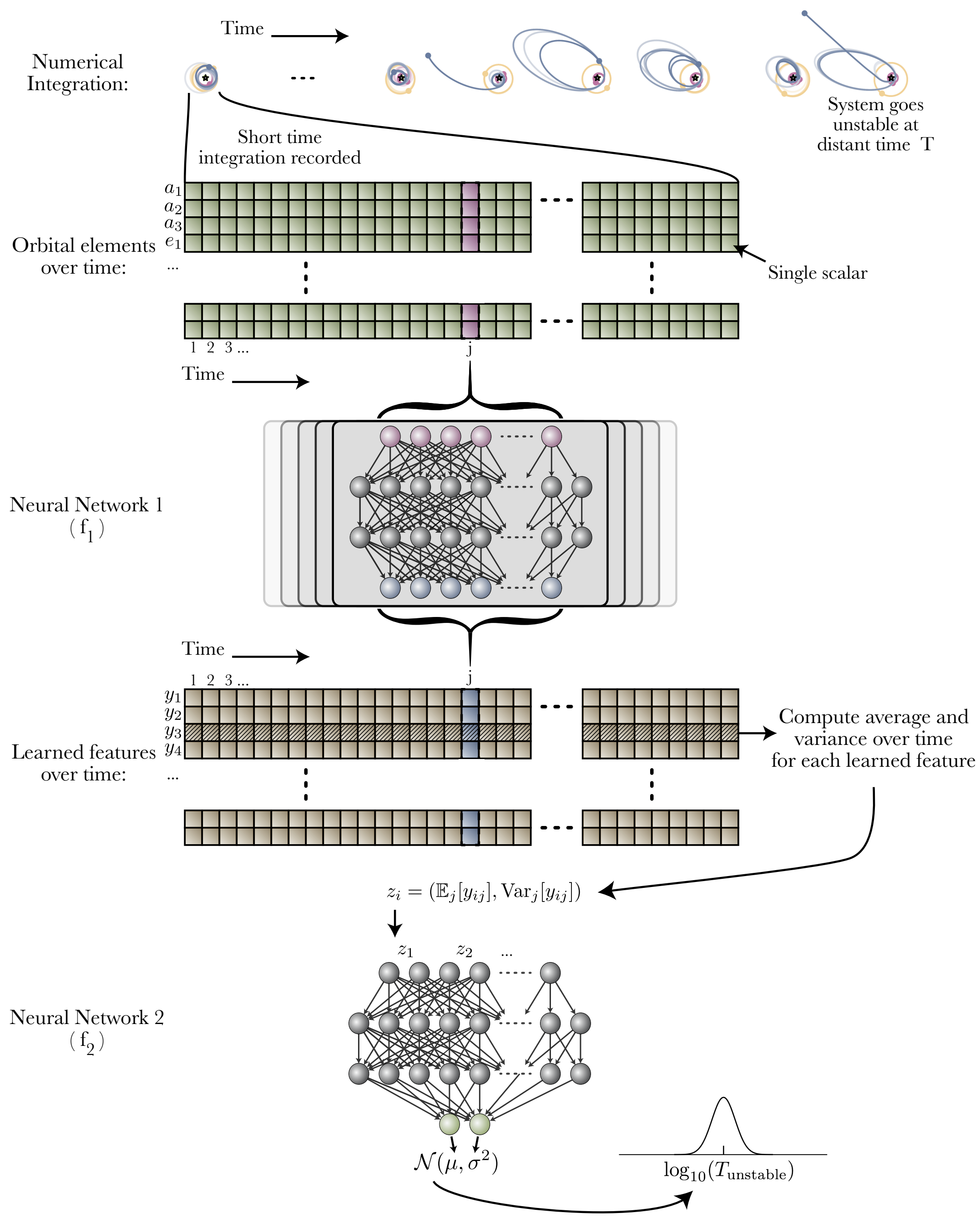
Cranmer et al (2021), Tamayo et al. (2020)

Step 3: Predicting time of instability



Cranmer et al (2021), Tamayo et al. (2020)

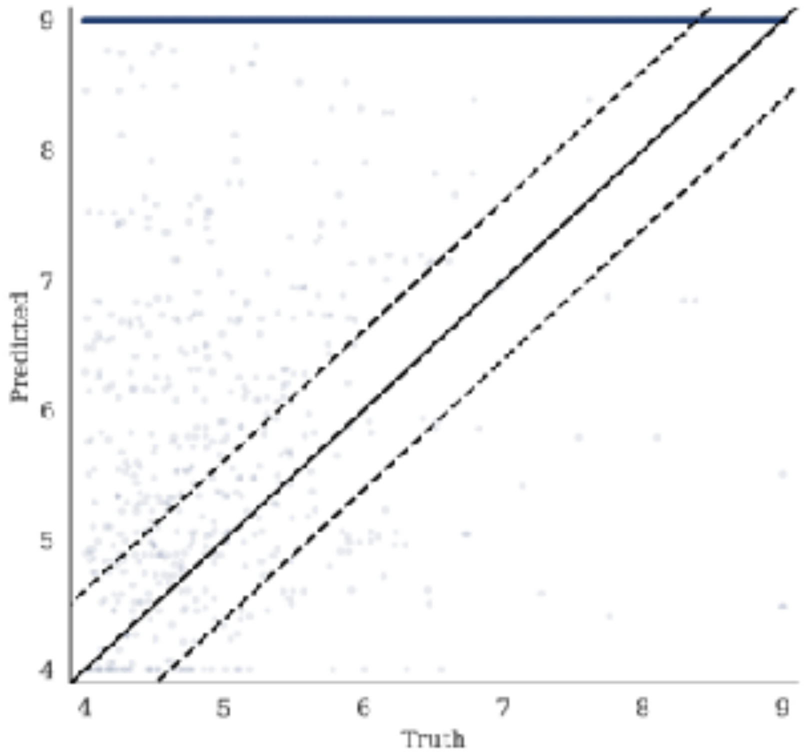
Bayesian Neural Network



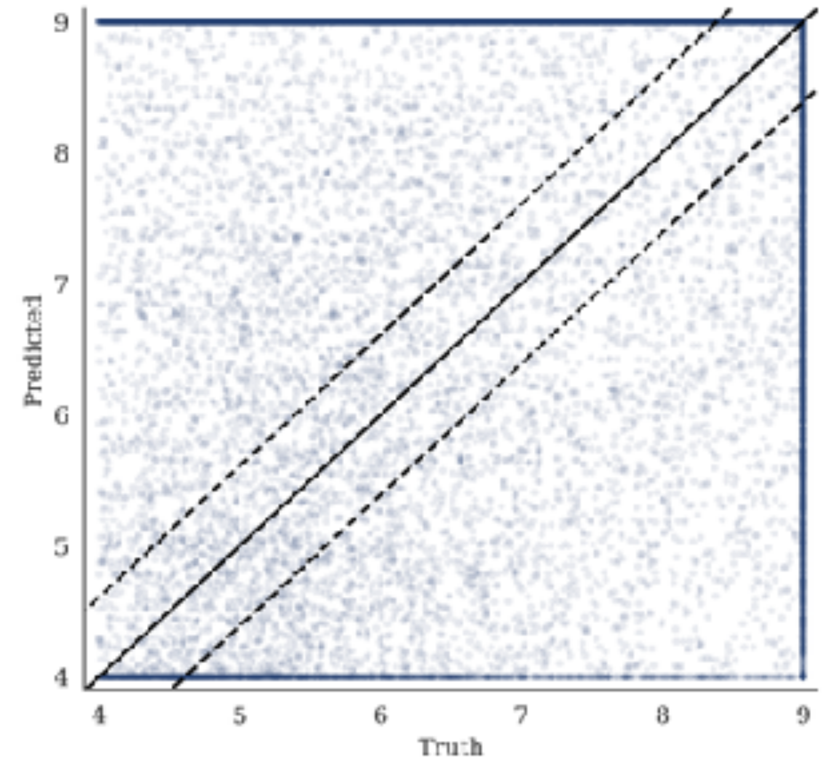
- Large training dataset: 113,543 simulation
- Generating training dataset is tricky
- Sampling a highly complex parameter space
- Computationally very expensive, billion orbits

Cranmer et al (2021), Tamayo et al. (2020)

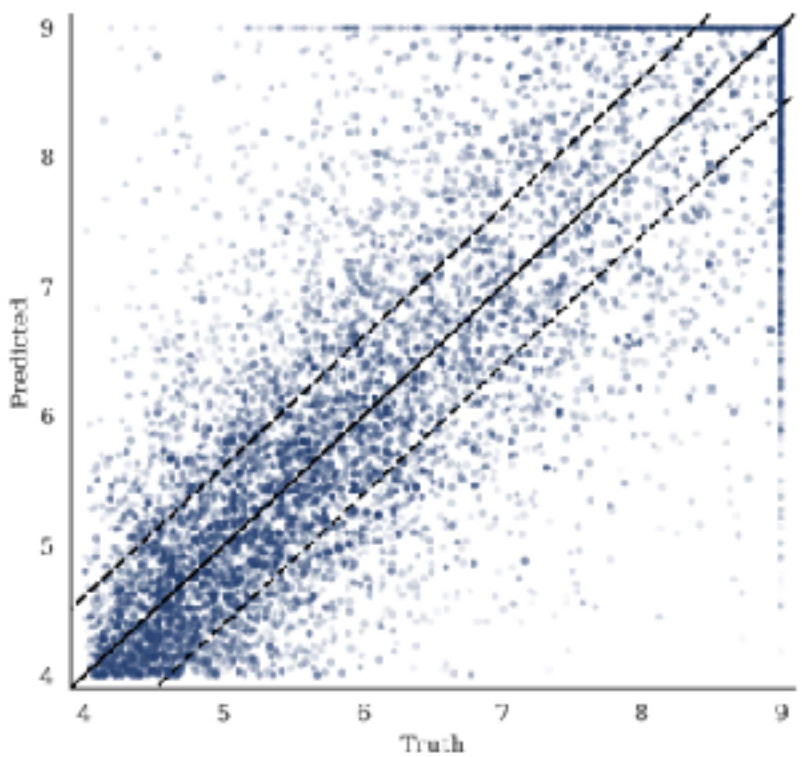
Results



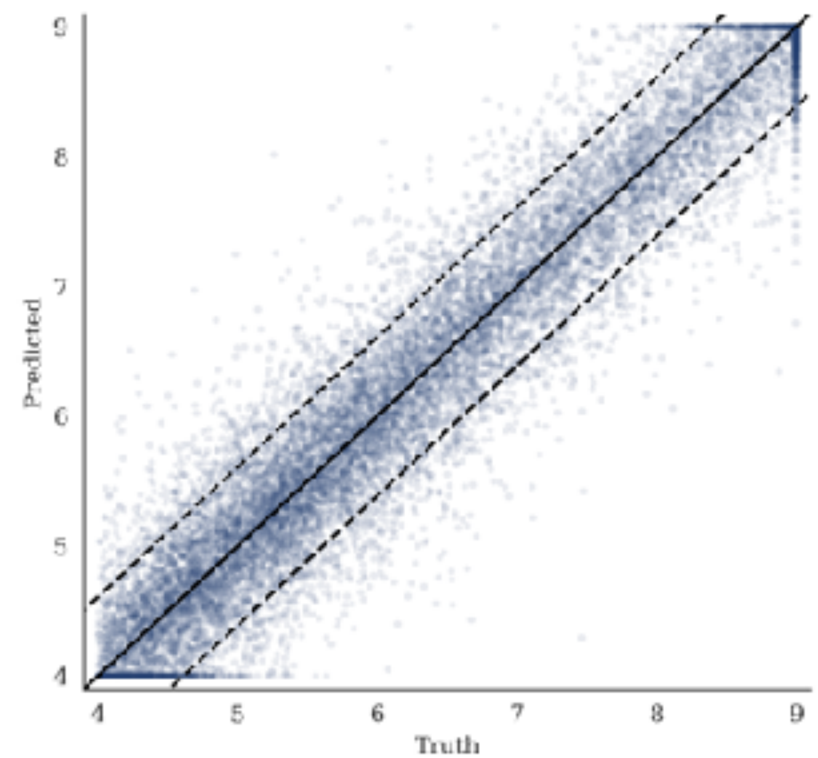
Petit et al. (2020)



Obertas et al. (2017)

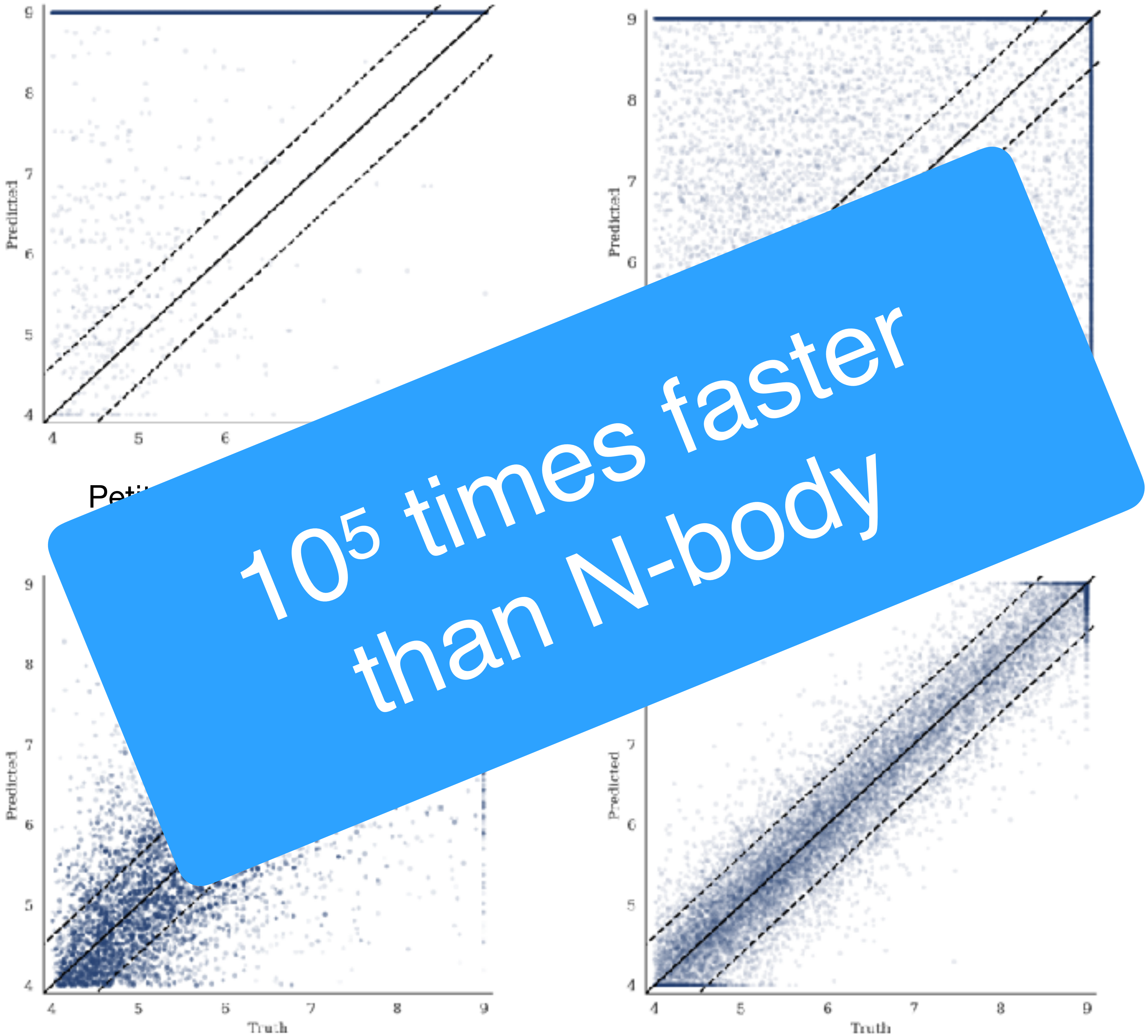


Our ML model, Cranmer et al (2021)



Theoretical limit

Results



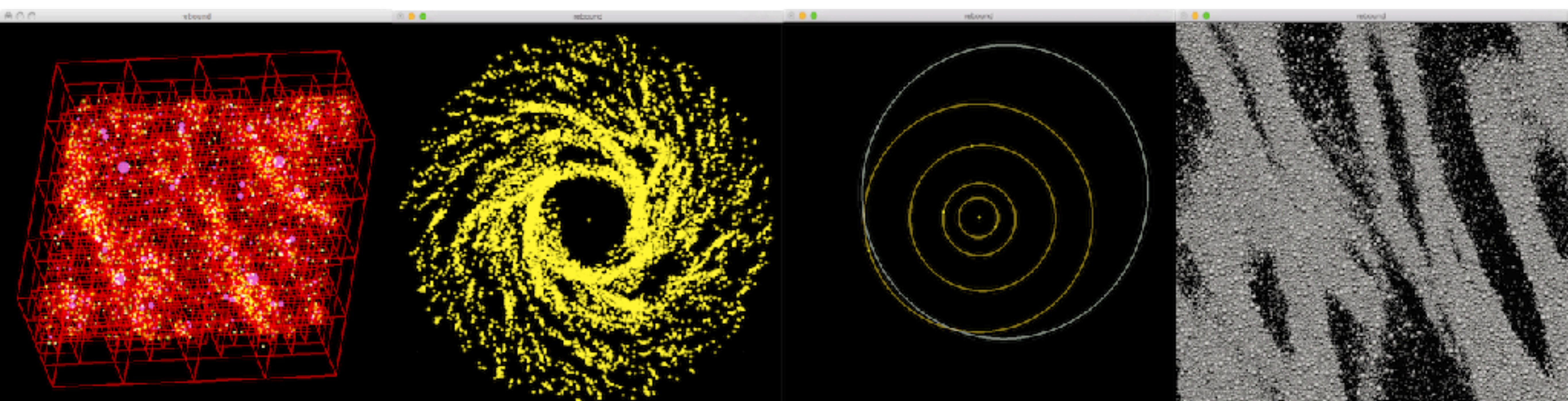
Our ML model, Cranmer et al (2021)

Theoretical limit

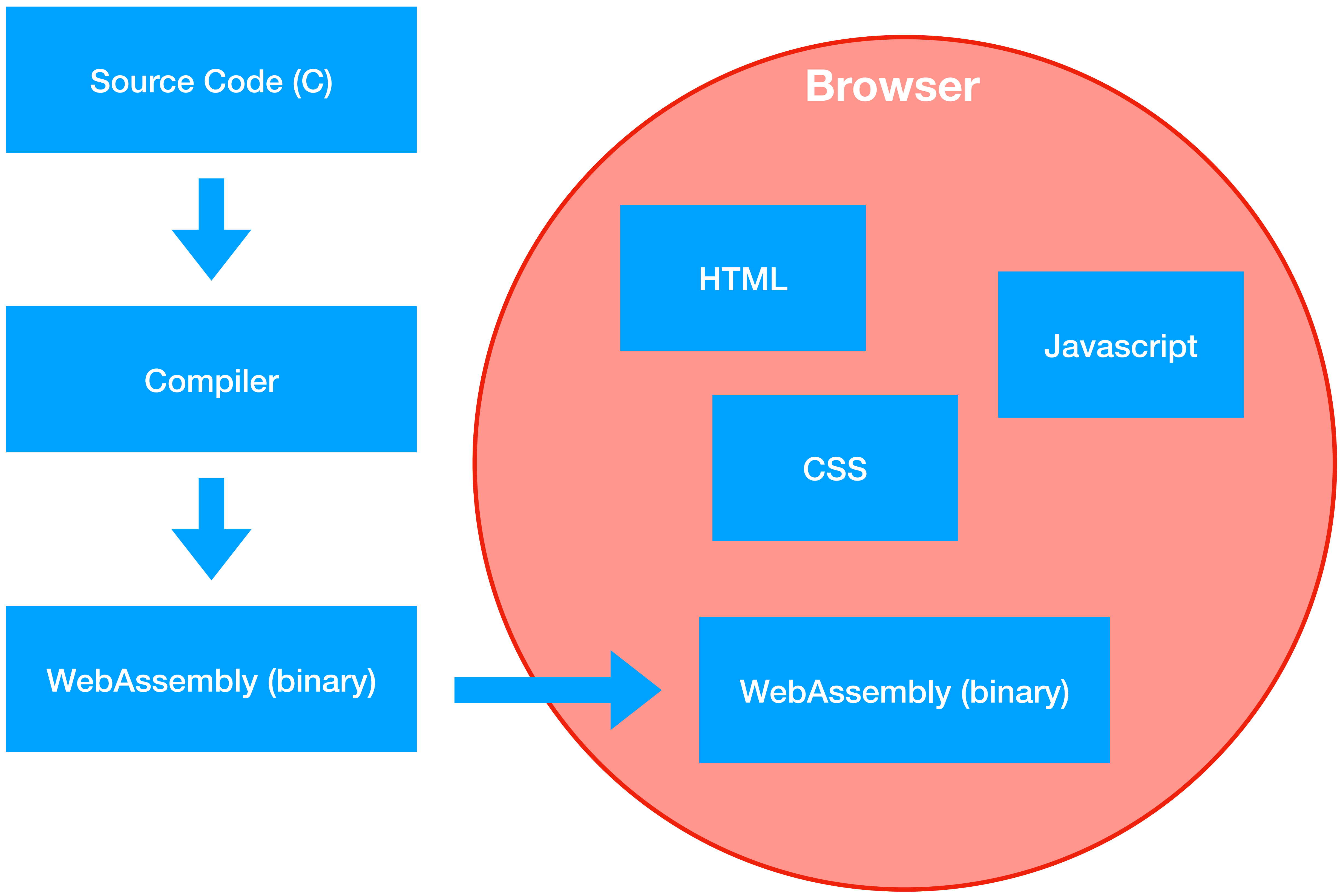
REBOUND, REBOUNDx, ASSIST

REBOUND

- N-body integrator package
- Many different built-in integrators
- Planetary systems
- Collisional simulations of planetary rings
- Written in C with an easy to use python interface
- No dependencies



WebAssembly



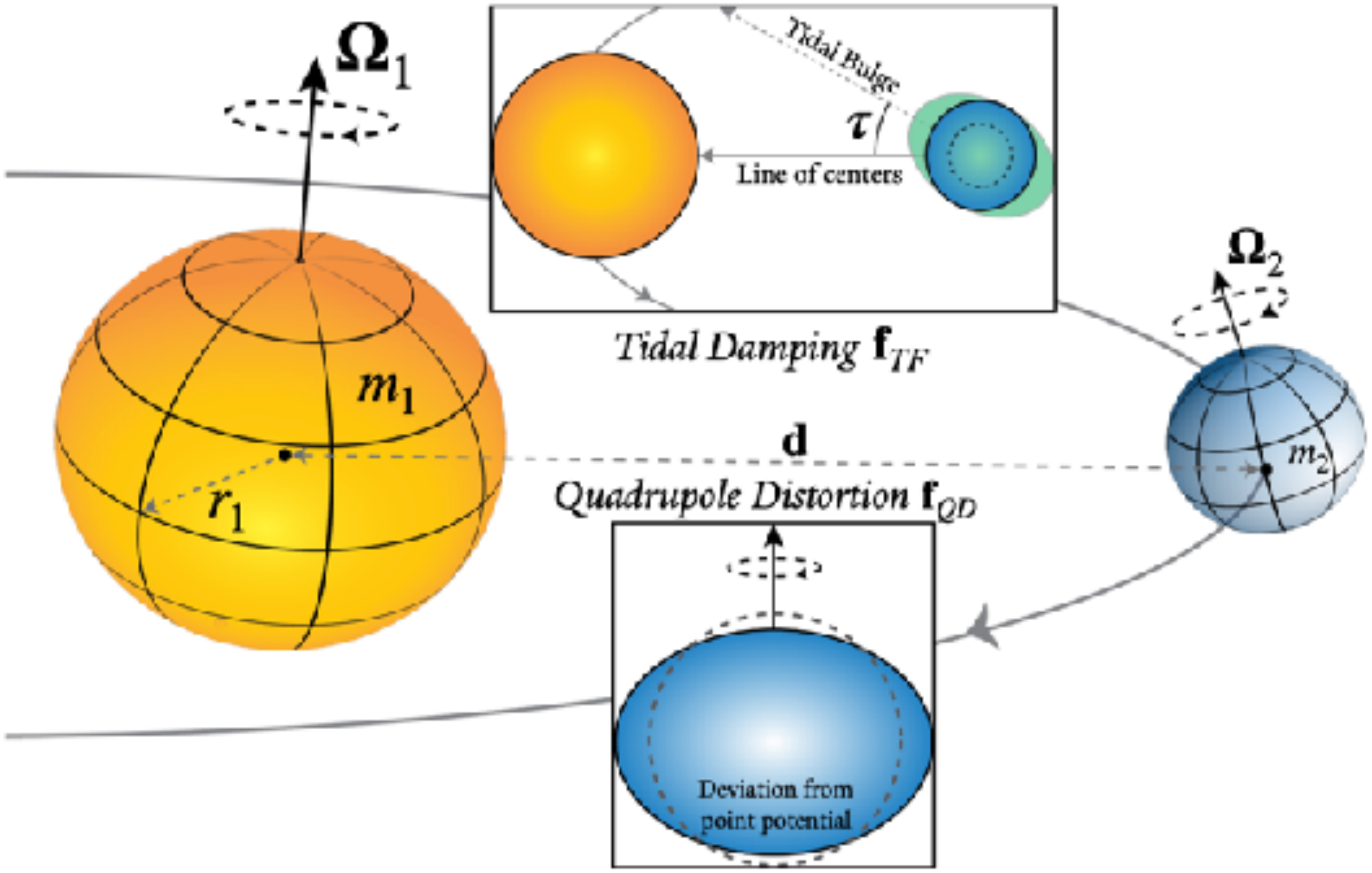
REBOUNDx Add-on

- Development led by Dan Tamayo (Harvey Mudd)
- Incorporate additional physics into N-body simulations:
    - Orbit modifications/migration
    - General relativity
    - Radiation pressure, Yarkovsky effect
    - Gravitational harmonics
    - Tides
- Very easy to use
- Does a lot of smart things behind the scenes!

Tamayo et al (2020)

REBOUNDx Tides

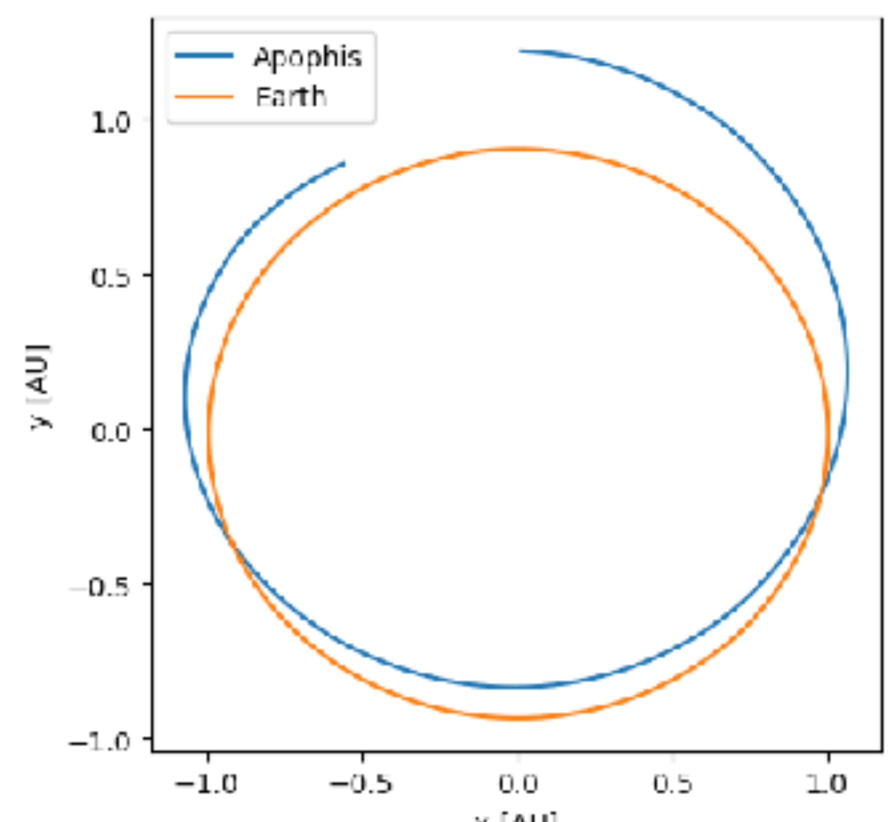
- Self-consistent spin, tidal and dynamical equations of motion
- Constant time lag approximation
- Part of REBOUNDx

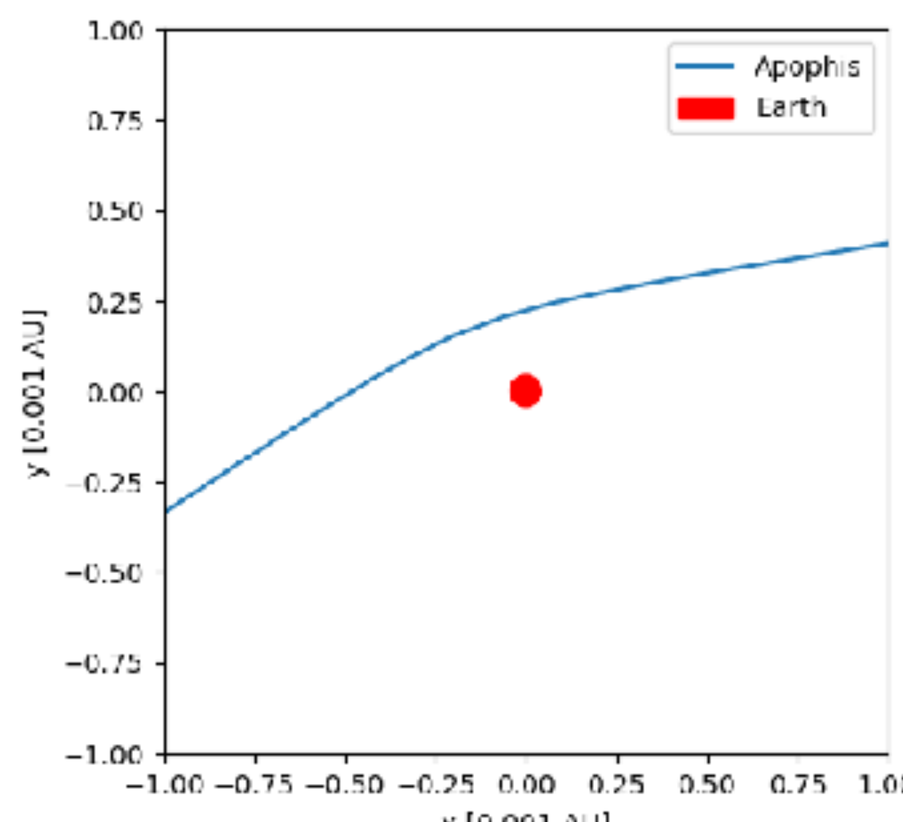


Lu et al (2023)

ASSIST Add-on

- Development led by Matt Holman (Minor Planet Center)
- Integration of asteroids, spacecrafts, artificial satellites in gravitational field of sun + planets (DE440 ephemeris)
- GR, radiation forces, higher order harmonics
- **Very** high accuracy, ~cm





Holman et al (2023, submitted)

Conclusions

Determining the stability of planetary systems is a very old problem. Analytic solutions cannot answer all question.

Chaos leads to collisional trajectories in the Solar System.

Embedded Operator Splitting methods (EOS) are very easy to implement. Can be configured to be equivalent to: leap-frog, Wisdom-Holman, Mercury, SYMBA, and many new methods.

Our machine-learning classifier can predict the stability of planetary systems 10^5 times faster.

Use the REBOUND/REBOUNDx/ASSIST ecosystem for all your small N dynamics needs.

Light pollution and mega constellations

Samantha M. Lawler, Aaron C. Boley, Hanno Rein (2022)

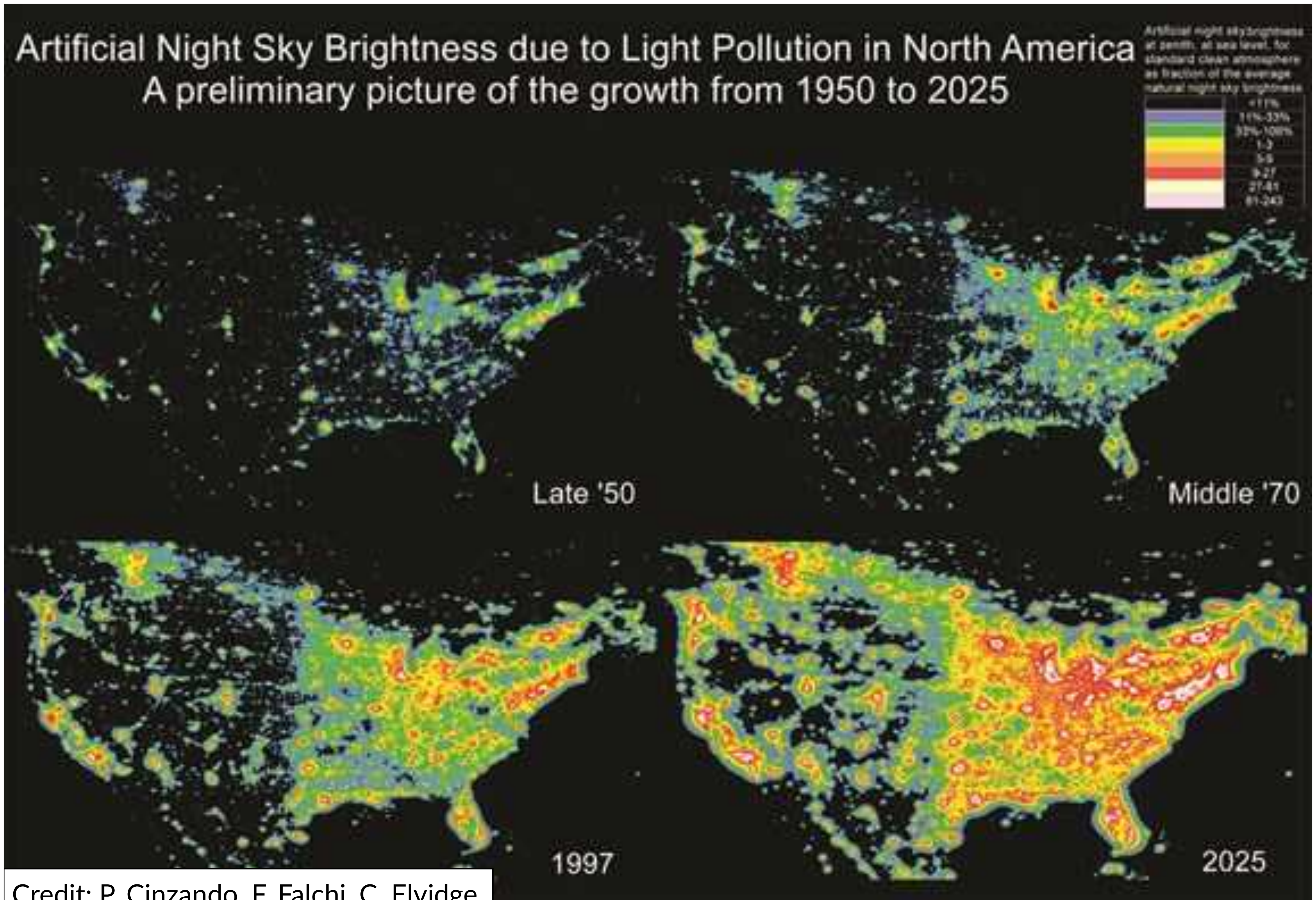

Plaskett Telescope

Aaron Boley, UBC

Samantha Lawler, U. of Regina

Hanno Rein, U. of Toronto

Access to the night sky is restricted by urban light pollution



Credit: P. Cinzando, F. Falchi, C. Elvidge

But there are many groups fighting!
(ex: International Dark Sky Association)

LEDs (a sudden leap in technology access) took these groups by surprise

LEDs are good – use less energy for more light. BUT are massively over-used because they're cheap.

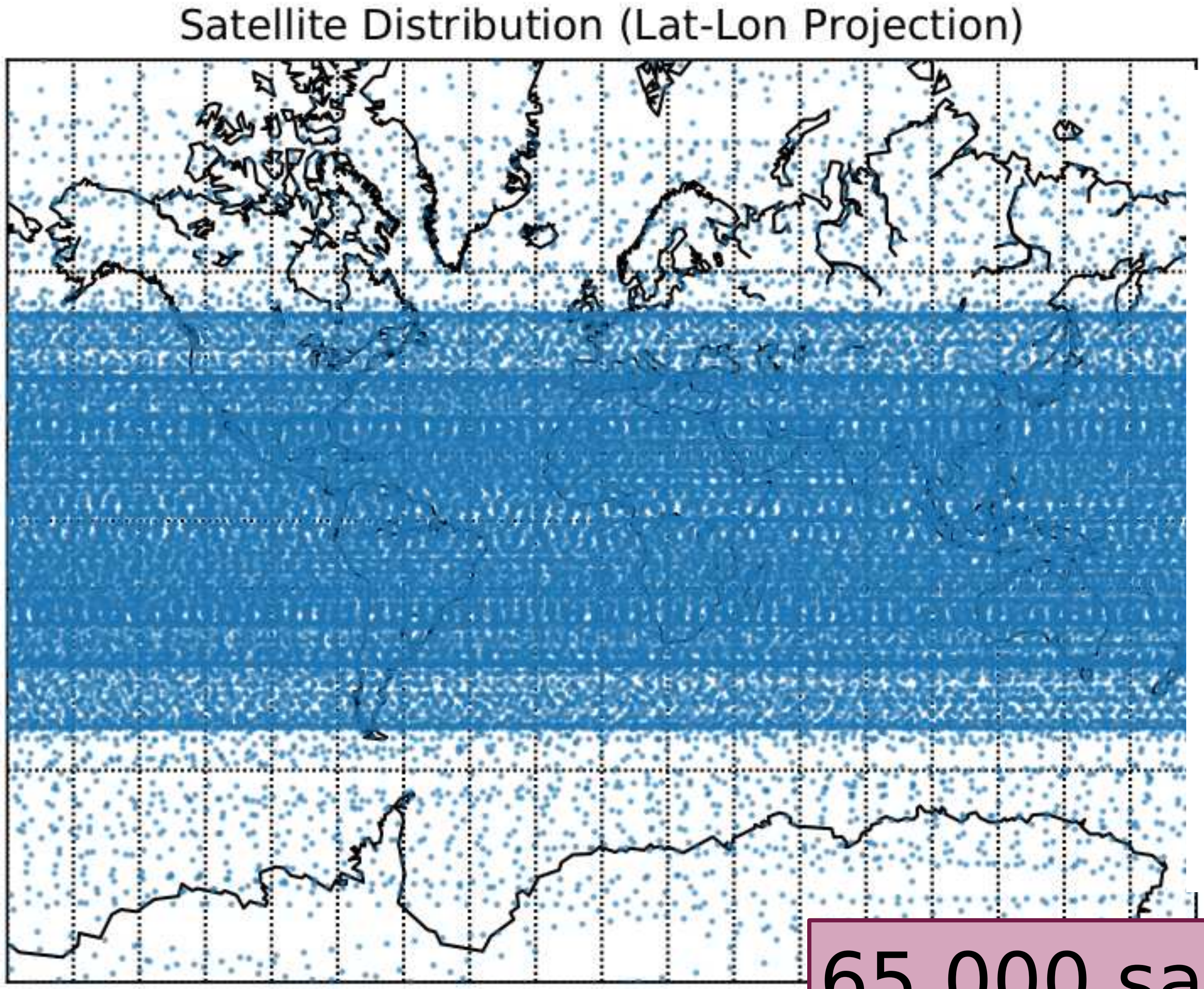
A new source of globally visible light pollution

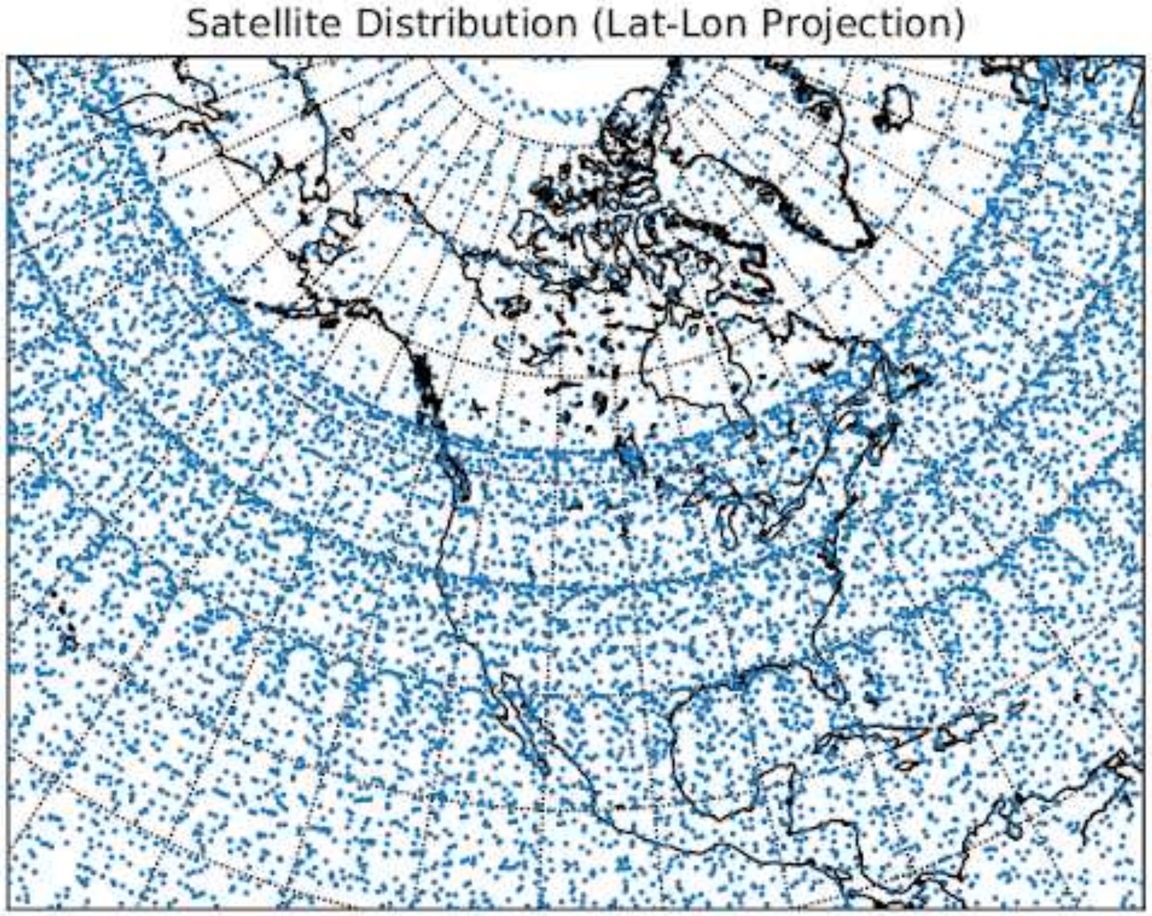
American private company SpaceX is launching batches of 60 satellites into low-Earth orbit **every 2-3 weeks**.

There are currently 3,633 Starlink satellites in orbit (out of 3,930 so far launched)

[Numbers from Jonathan McDowell's Starlink Statistics Page up to date as of 16 Feb 2023]

How bad could it get?

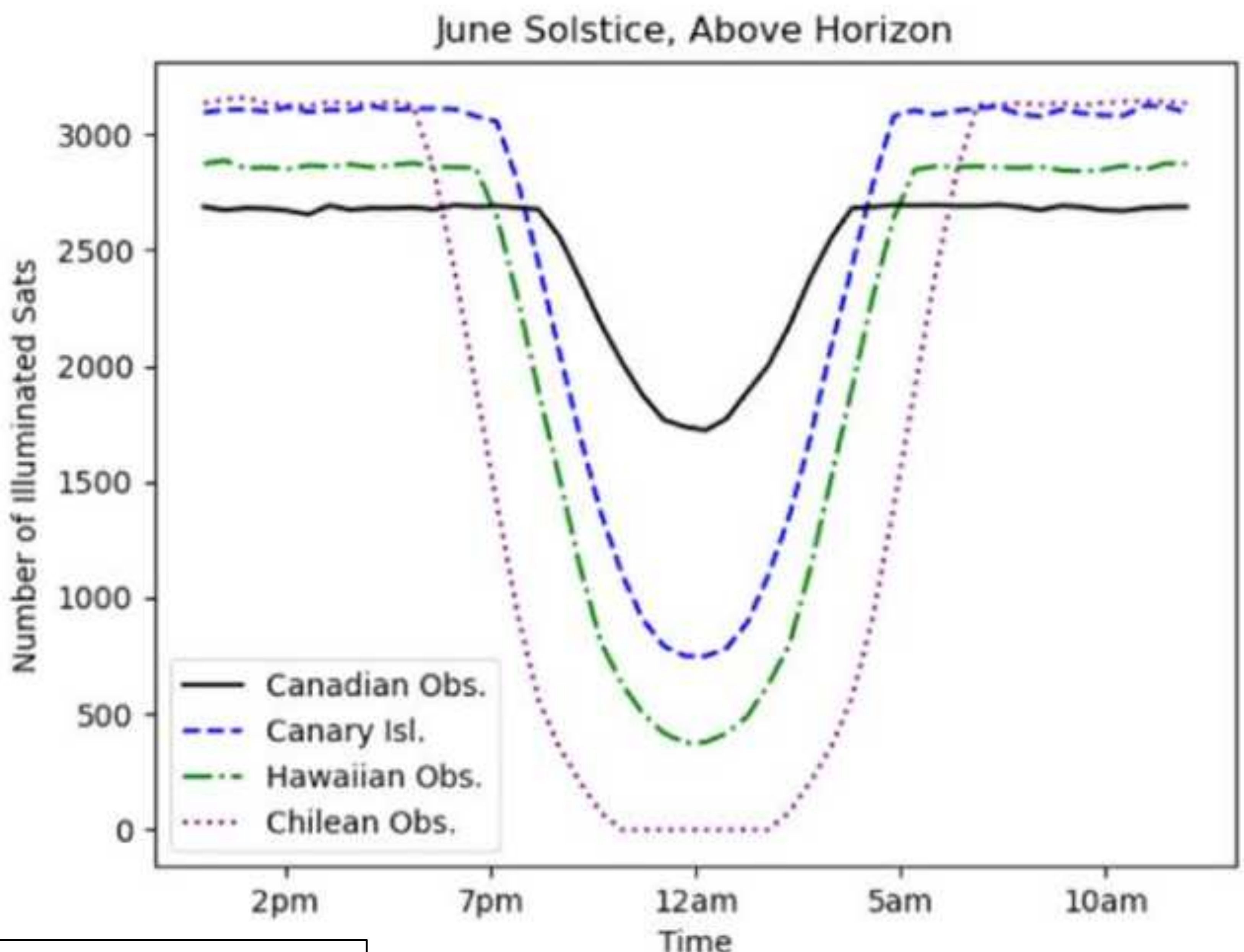




65,000 satellites

Lawler, Boley & Rein (2022)

How bad could it get?



Lawler, Boley & Rein (2022)

Straightforward to calculate how many satellites are above the horizon and illuminated by sunlight.

... But how bright the satellites are when illuminated in orbit depends entirely on unknown engineering.


Constellations
Constellations
Settings
Megaconstellations
Available for free on the AppStore
February 27
3:36 PM (UTC)
Perspective: Orbit
About