UNIVERSITY OF TORONTO

Simulating the Solar System for 10 Billion Years

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Collaborators

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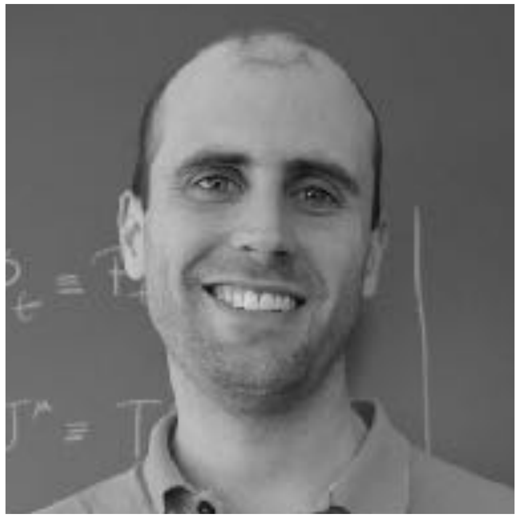
IAS, Princeton
Data Scientist at Stitch Fix



Dan Tamayo

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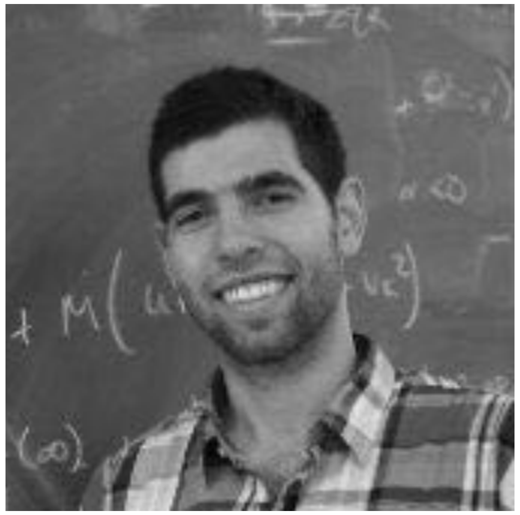
Aleksandar Rachkov

PhD student, University of Toronto



Ari Silburt

PhD student, University of Toronto

Physics

Numerics

Gravity

$$\ddot{\mathbf{r}}_i = \sum_{\substack{j=1 \\ j\neq i}}^{N} m_j \frac{\mathbf{r}_j - \mathbf{r}_i}{\left|\mathbf{r}_j - \mathbf{r}_i\right|^3}$$

Newton (1687)

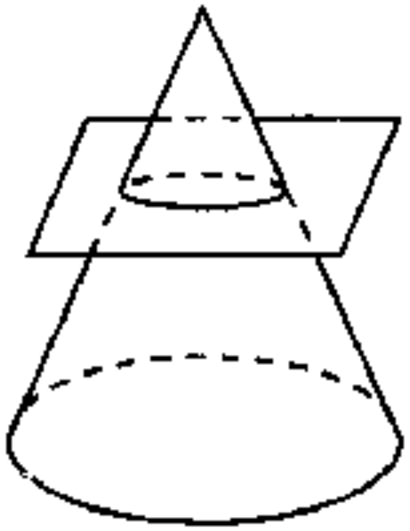
1-body problem

$$\ddot{\mathbf{r}}(t) = \mathbf{0}$$

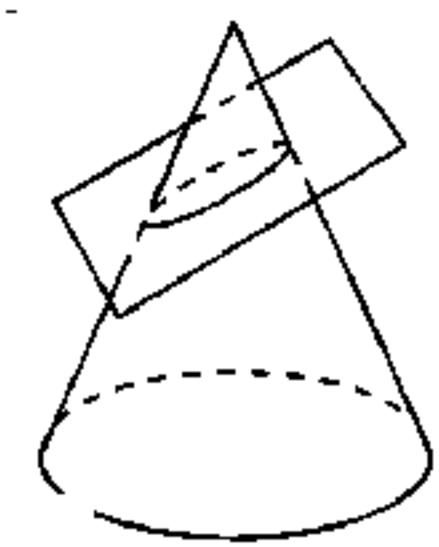
$$\dot{\mathbf{r}}(t) = \mathbf{v}_0$$

$$\mathbf{r}(t) = \mathbf{r}_0 + t\ \mathbf{v}_0$$

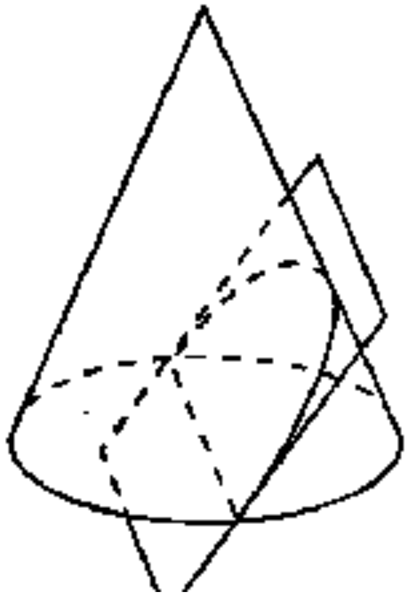
2-body problem



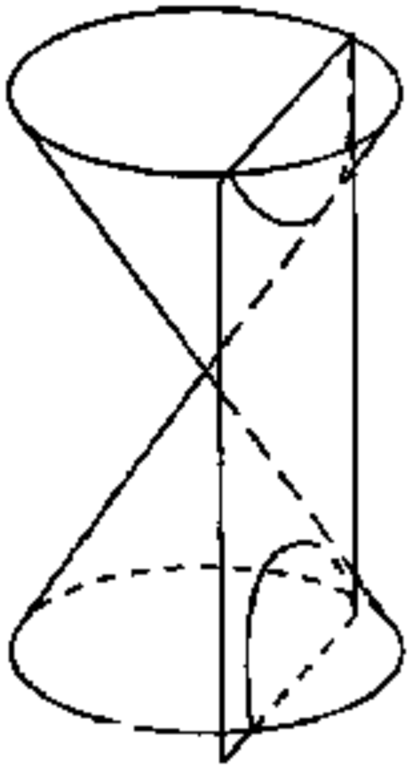
Circle



Ellipse



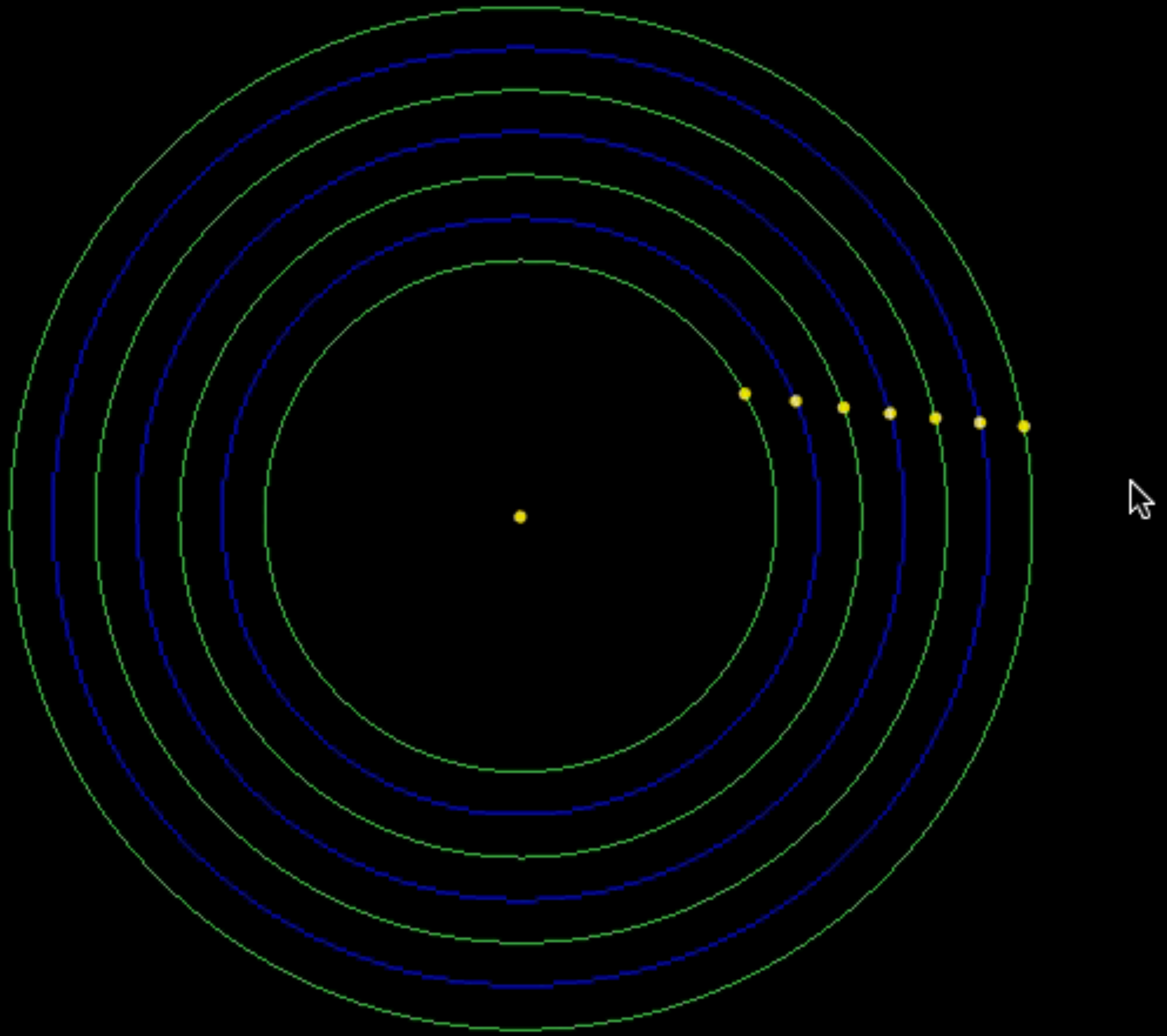
Hyperbola



Parabola

N-body problem?

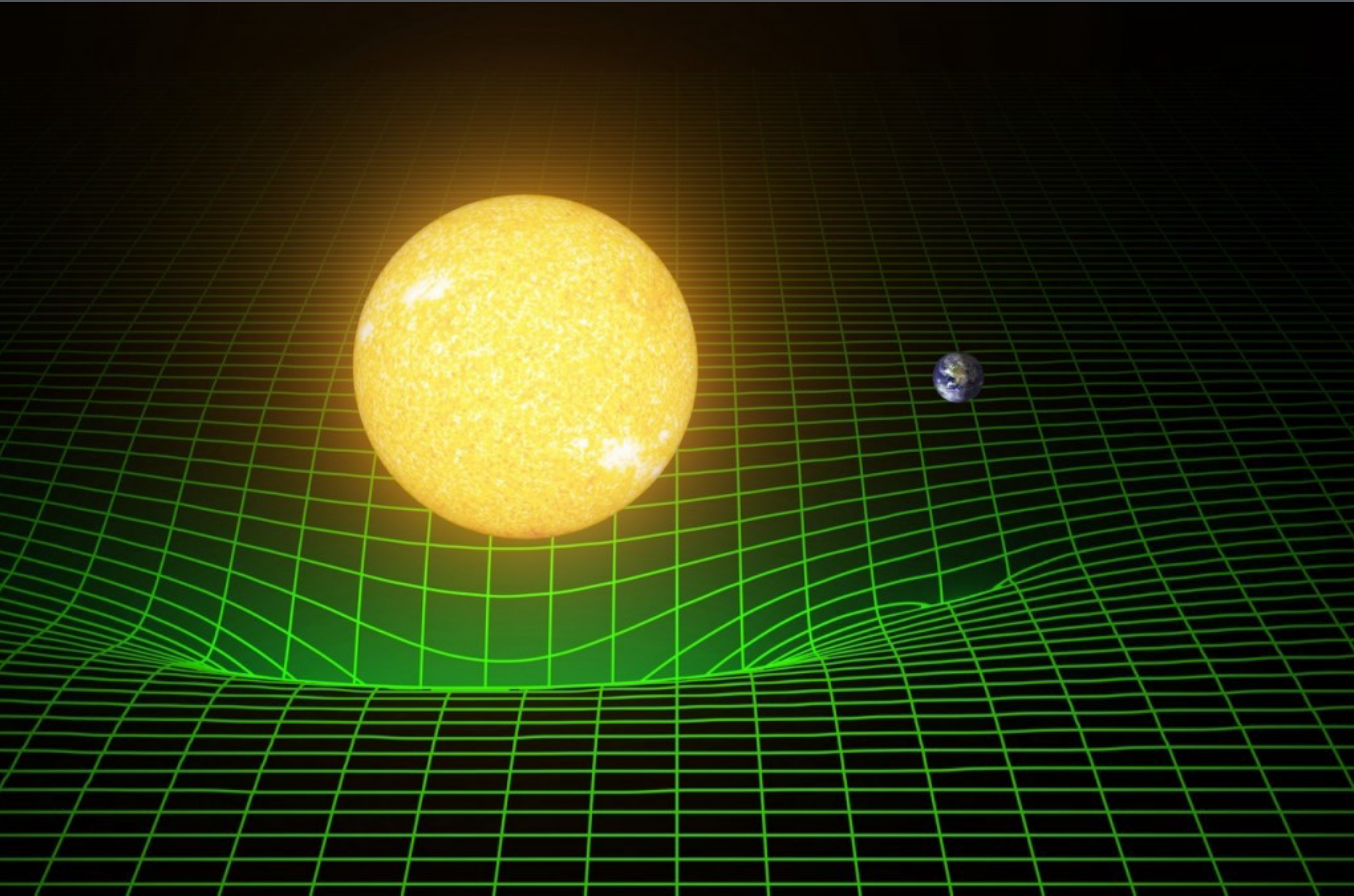
N-body problem



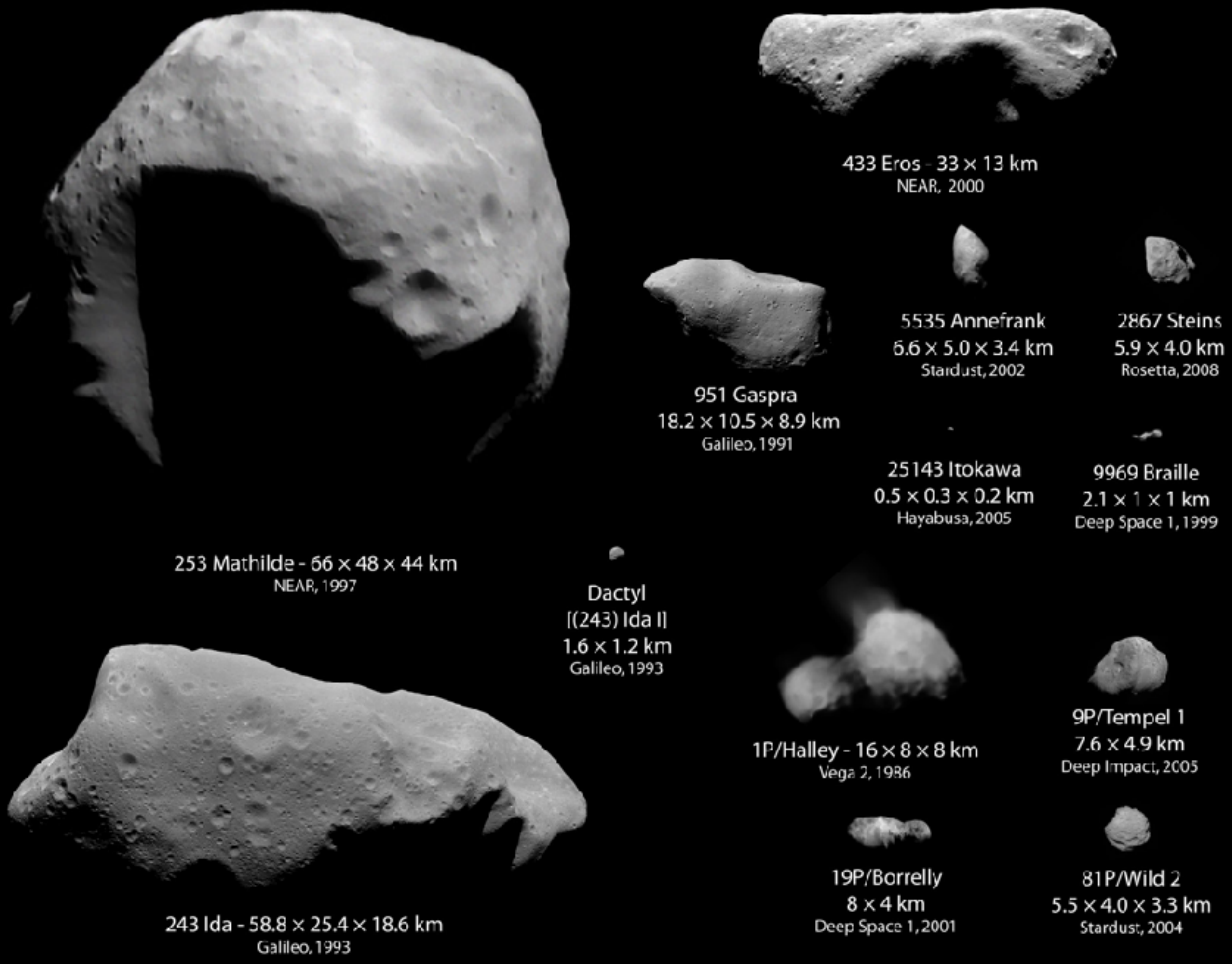
Long term integrations of the Solar System

Physical effects besides Newtonian gravity

(Full) General relativity

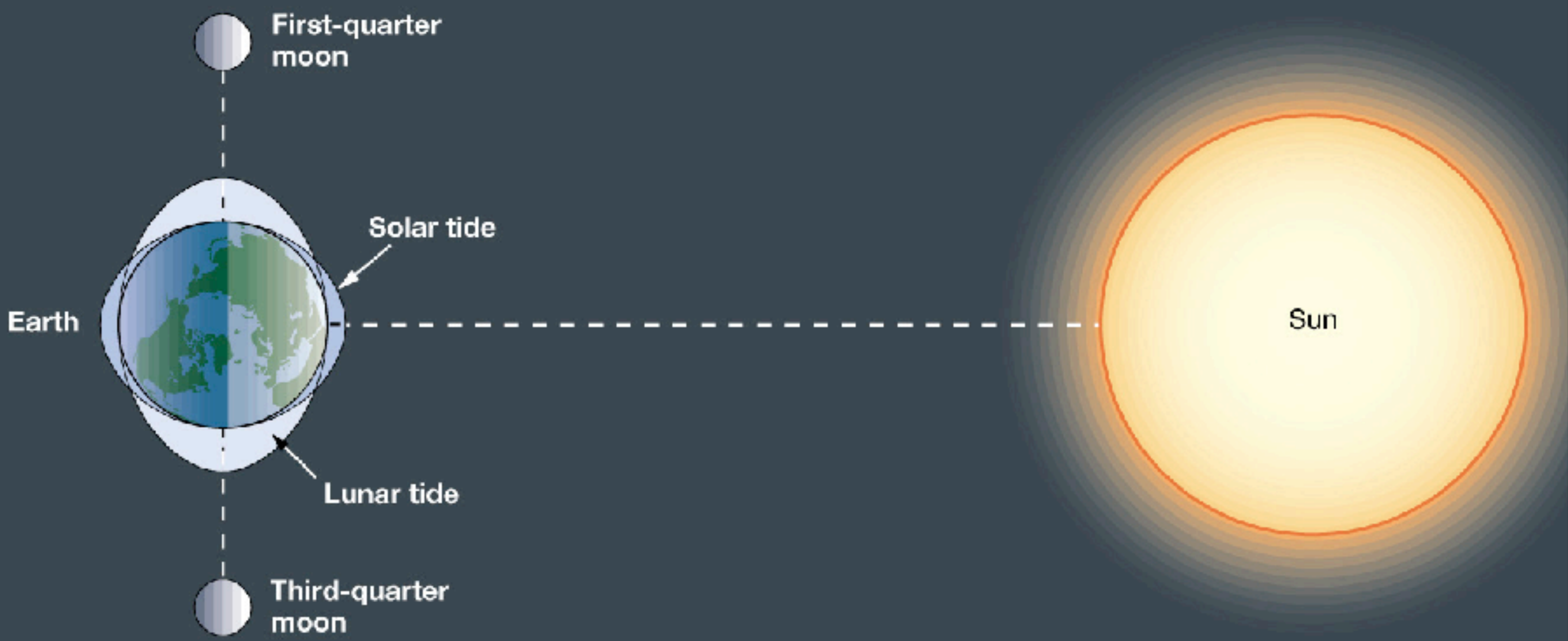


Asteroids

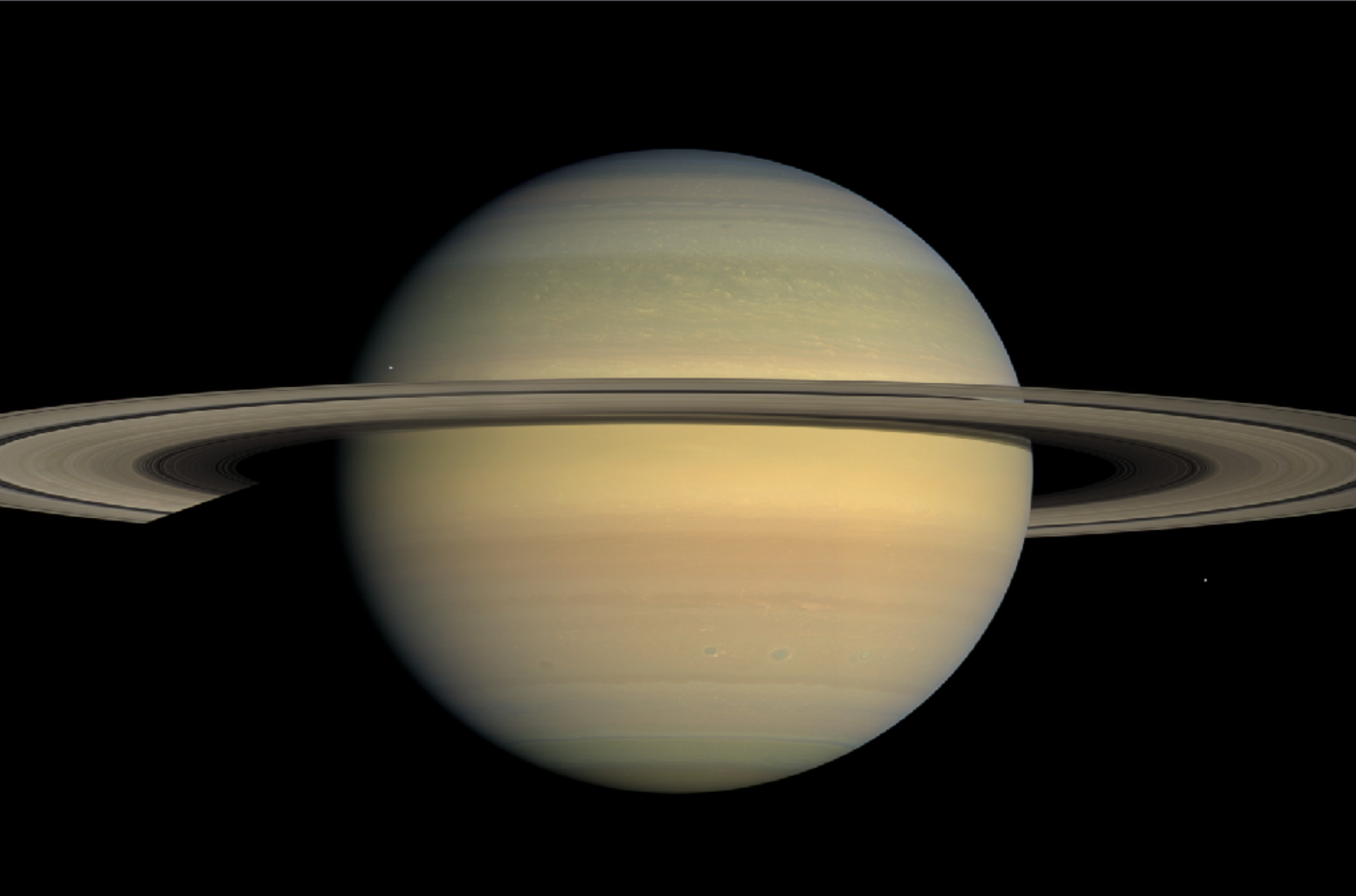

433 Eros - 33 × 13 km
NEAR, 2000
951 Gaspra
18.2 × 10.5 × 8.9 km
Galileo, 1991
5535 Annefrank
6.6 × 5.0 × 3.4 km
Stardust, 2002
2867 Steins
5.9 × 4.0 km
Rosetta, 2008
25143 Itokawa
0.5 × 0.3 × 0.2 km
Hayabusa, 2005
9969 Braille
2.1 × 1 × 1 km
Deep Space 1, 1999
253 Mathilde - 66 × 48 × 44 km
NEAR, 1997
Dactyl
[(243) Ida I]
1.6 × 1.2 km
Galileo, 1993
1P/Halley - 16 × 8 × 8 km
Vega 2, 1986
9P/Tempel 1
7.6 × 4.9 km
Deep Impact, 2005
19P/Borrelly
8 × 4 km
Deep Space 1, 2001
81P/Wild 2
5.5 × 4.0 × 3.3 km
Stardust, 2004
243 Ida - 58.8 × 25.4 × 18.6 km
Galileo, 1993

Tides

(a) Spring tide

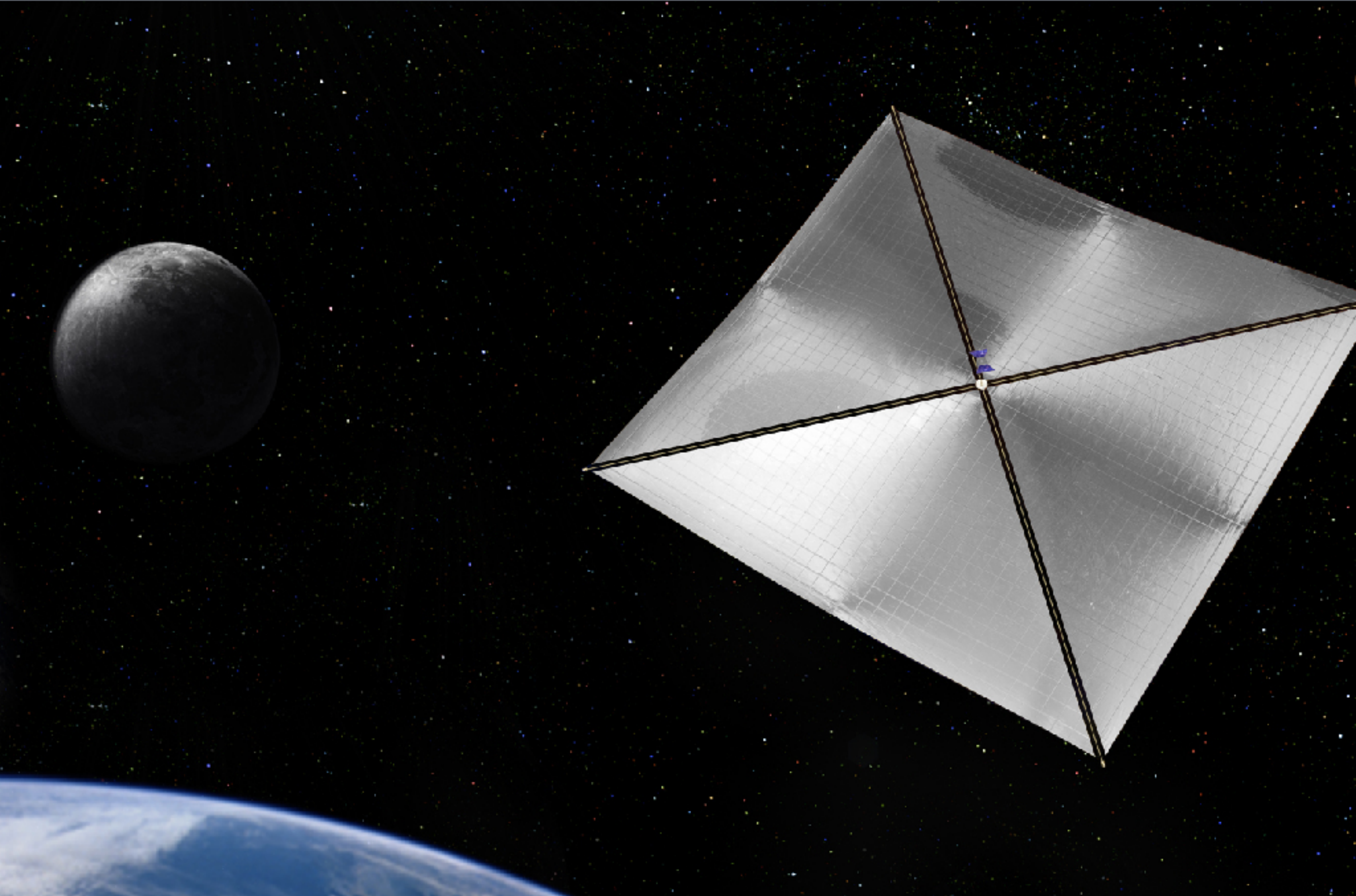


Quadrupole moments



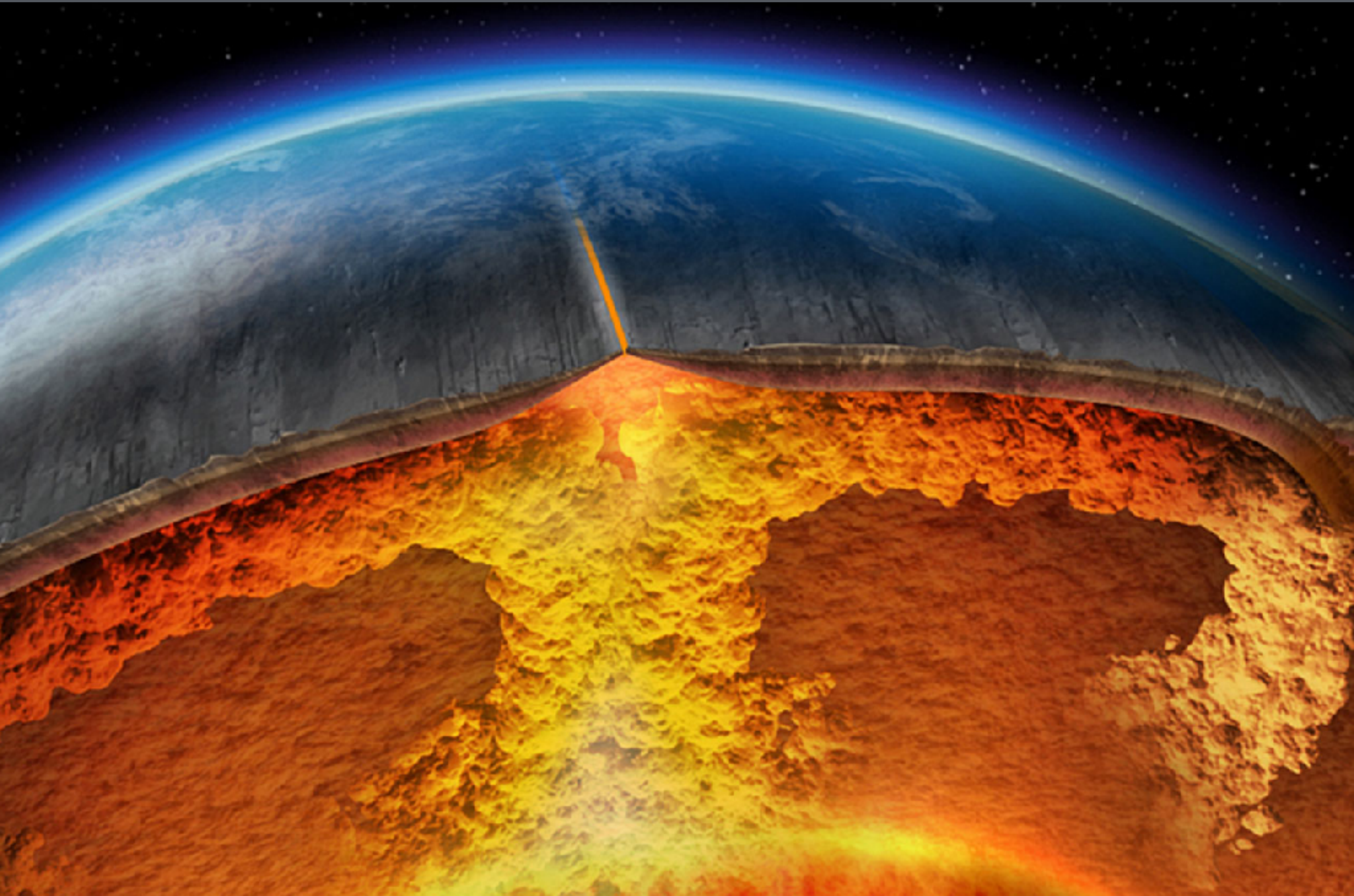
Solar mass loss

Solar wind/radiation drag



Galactic tidal forces

Earth mantel friction



Timescales

Timescales

short

Orbital period of Naiad

7 hours

Orbital period of Mercury

2 100 hours

Orbital period of Pluto

2 100 000 hours

(Quasi) Resonant / Secular interactions

100 000 000 hours

Lyapunov timescale

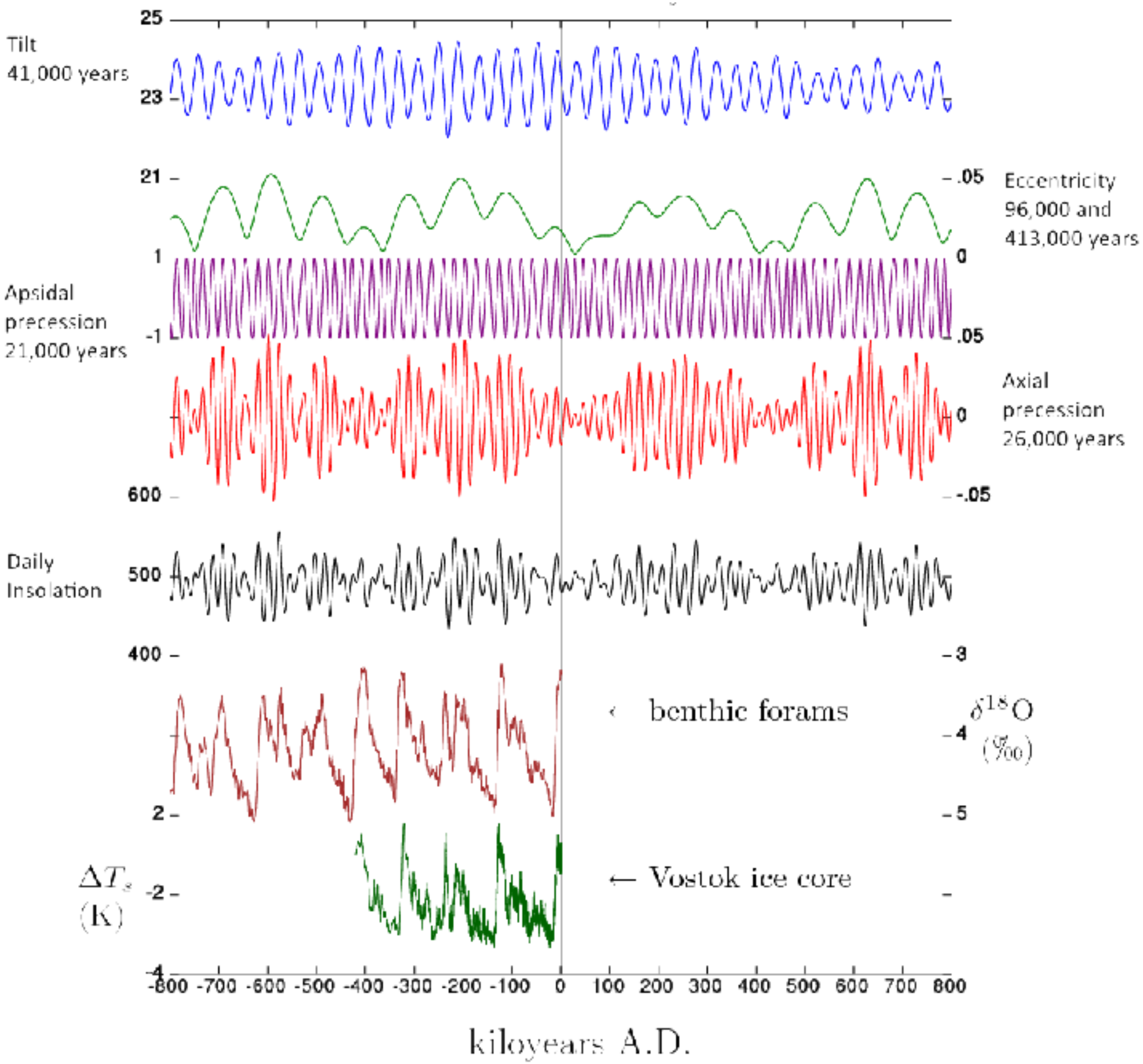
44 000 000 000 hours

Age of the Solar System

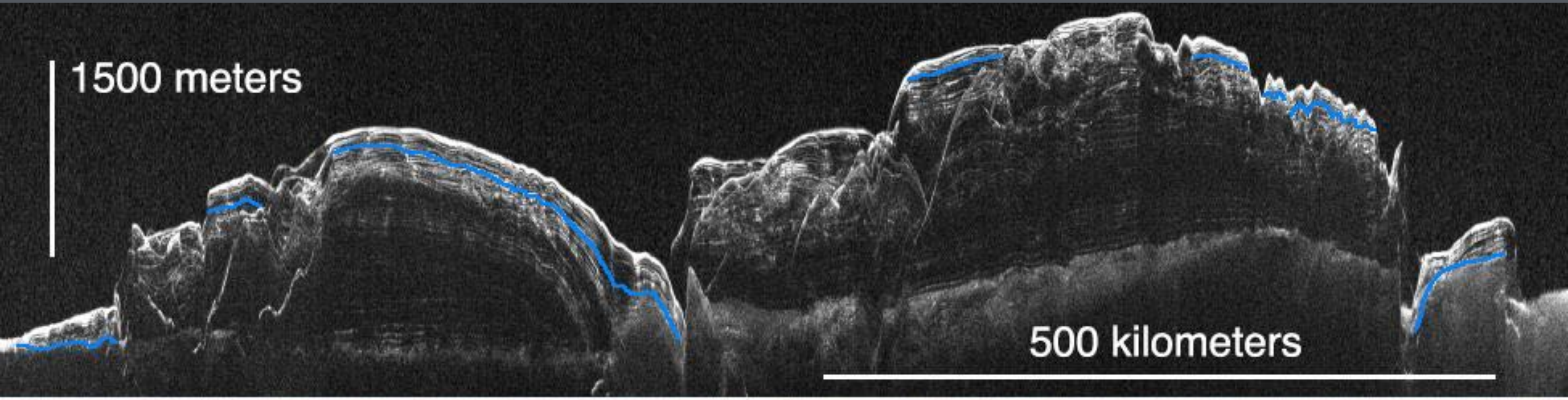
40 000 000 000 000 hours

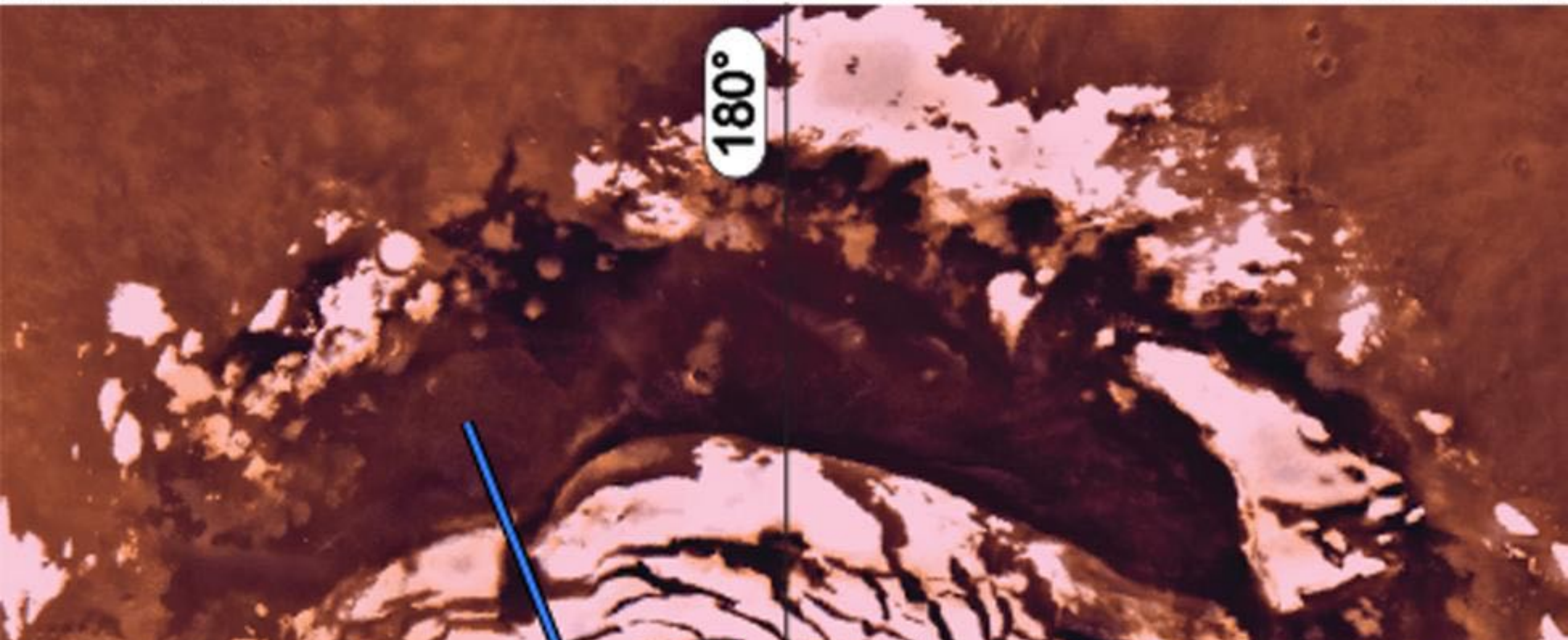
long

Milankovitch Cycle

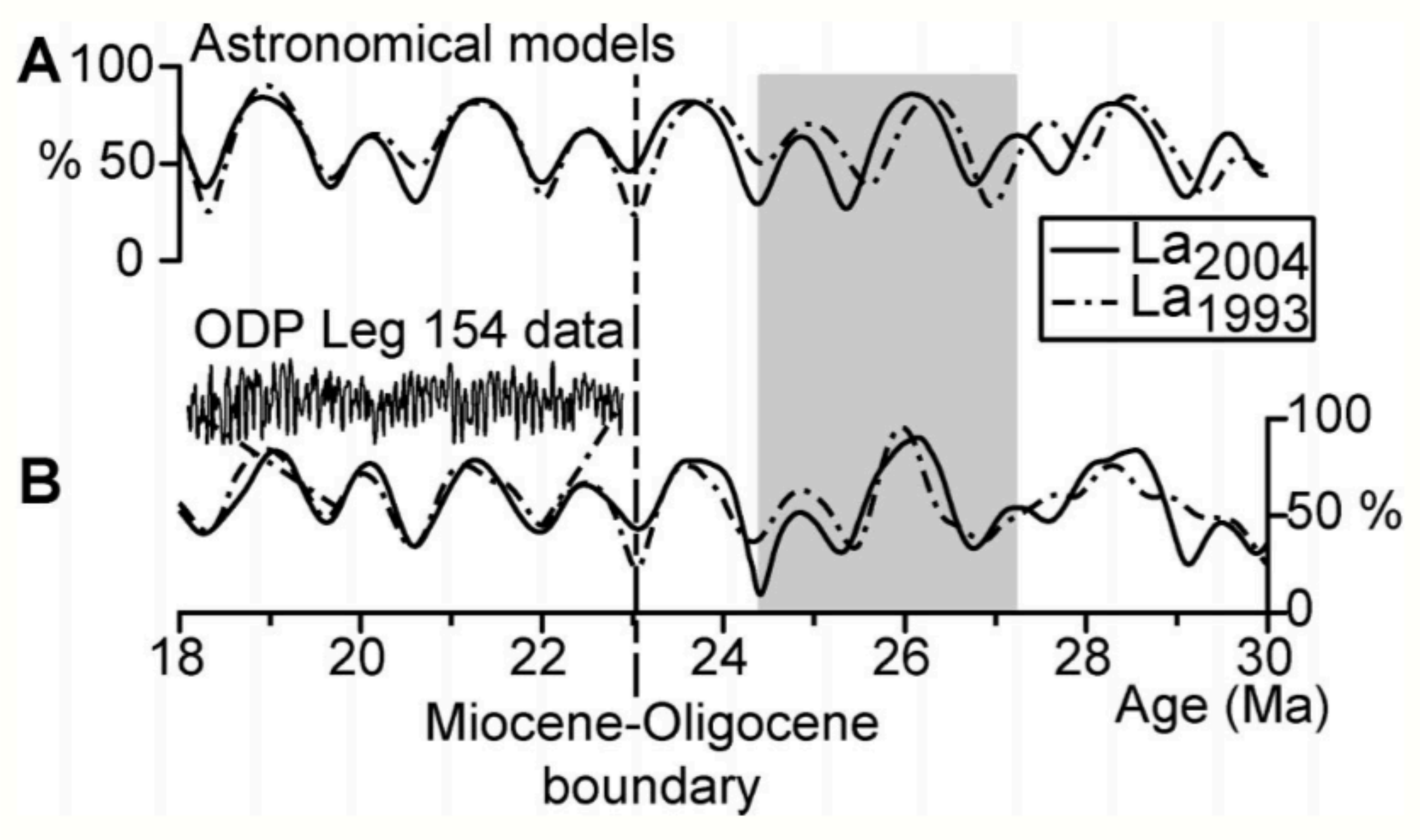


Milankovitch Cycle





Geologic constraints on chaotic diffusion



Pälike et al 2004

Earth’s obliquity

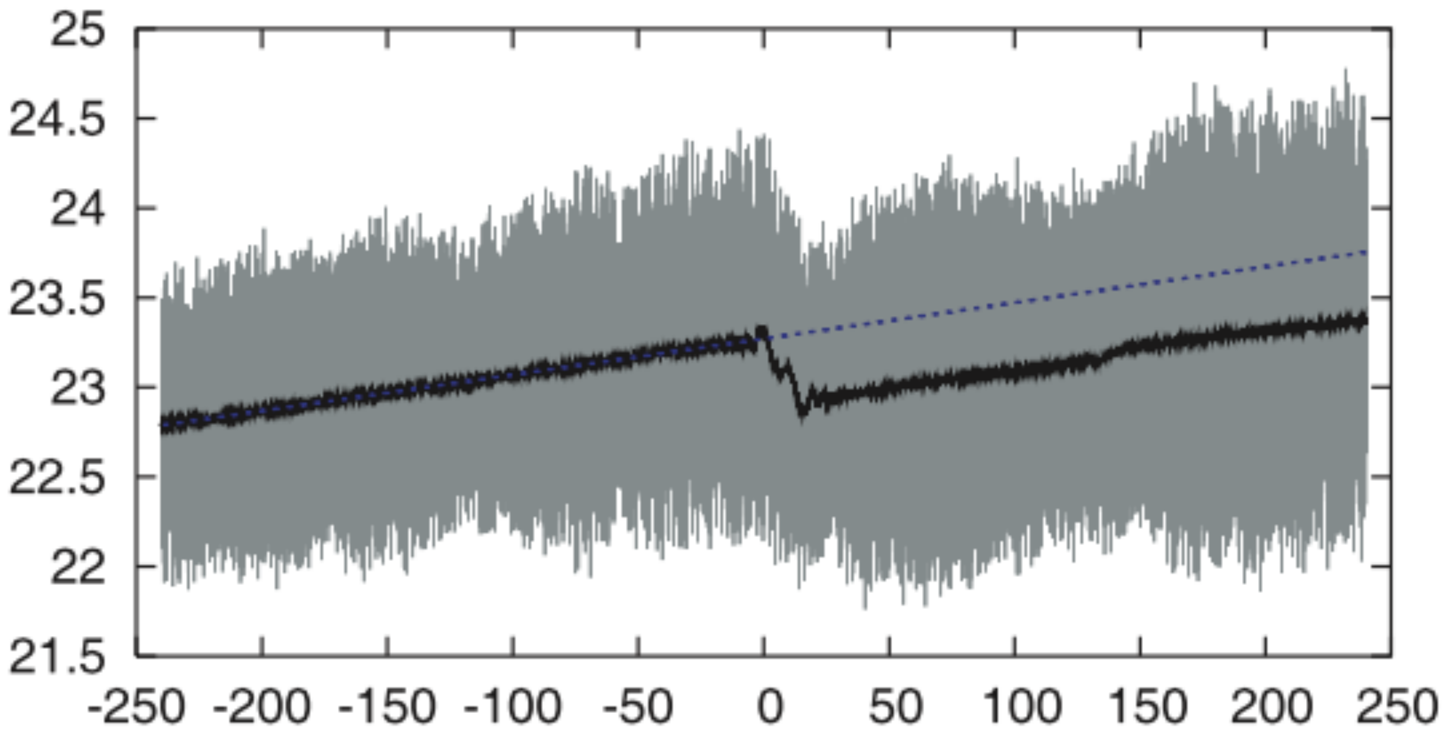
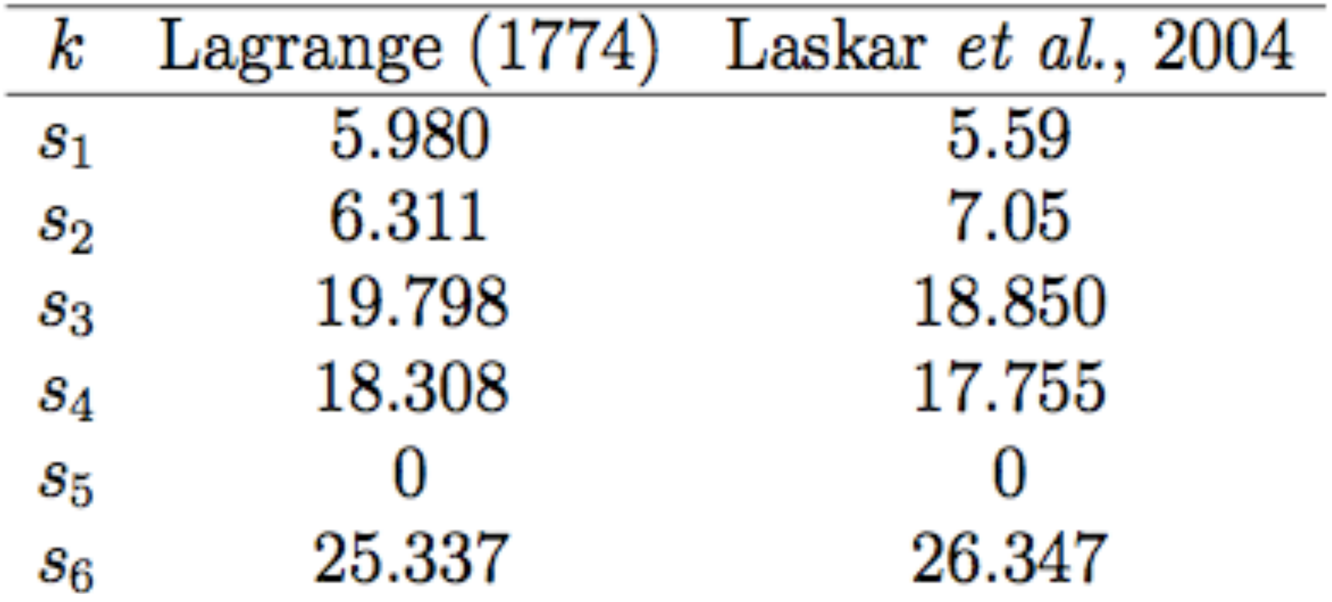


Fig. 14. Evolution of the obliquity of the Earth in degrees, from −250 to +250 Myr. The grey zone is the actual obliquity, while the black curve is the averaged value of the obliquity over 0.5 Myr time intervals. The dotted line is a straight line fitted to the average obliquity in the past.

Laskar et al 2013

History of secular perturbation theory

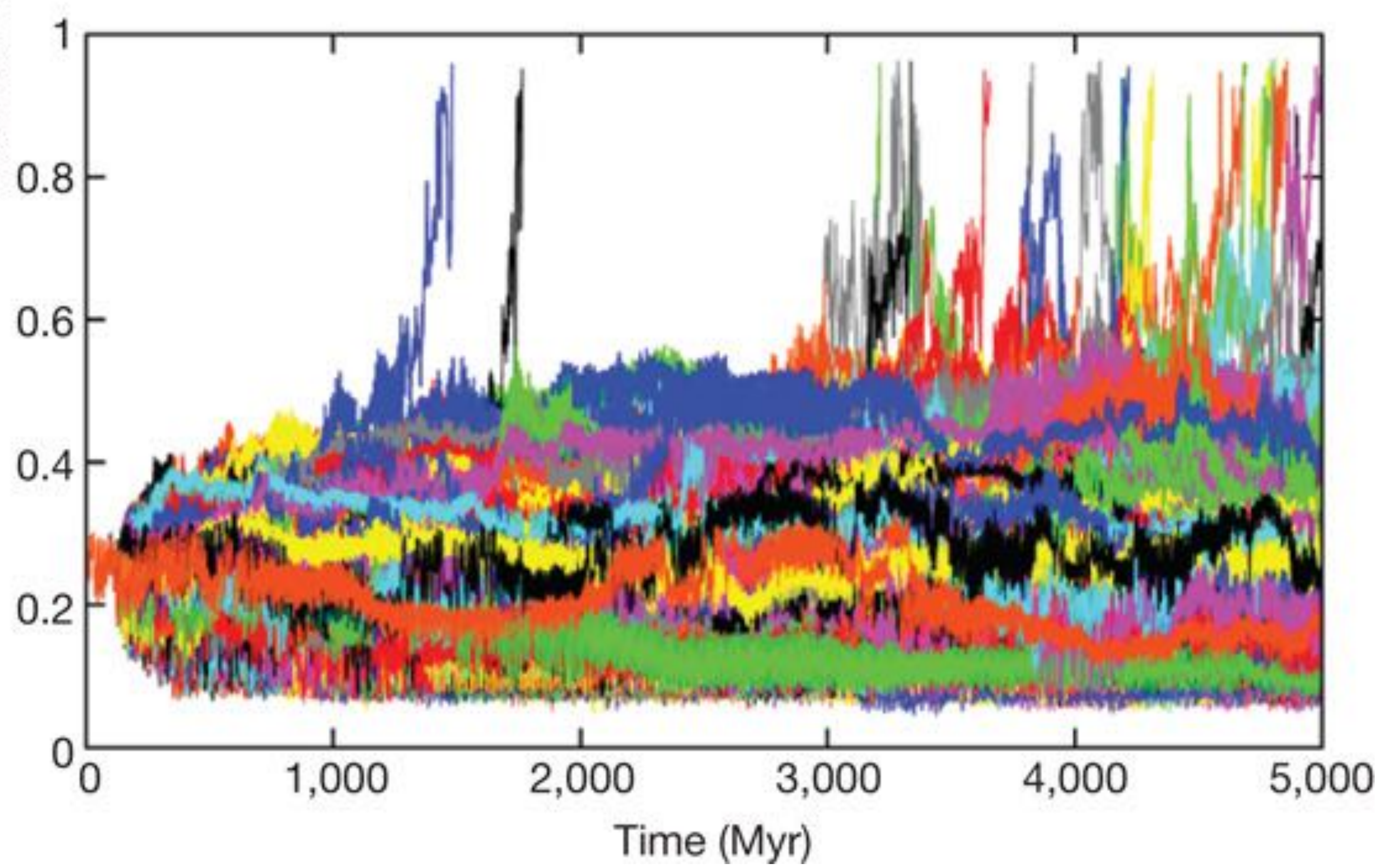


k	Lagrange (1774)	Laskar *et al.*, 2004
s_1	5.980	5.59
s_2	6.311	7.05
s_3	19.798	18.850
s_4	18.308	17.755
s_5	0	0
s_6	25.337	26.347

Laskar 2012

Hyperion

Mercury’s eccentricity



Laskar & Gastineau 2009

Collisional trajectories of the inner planets

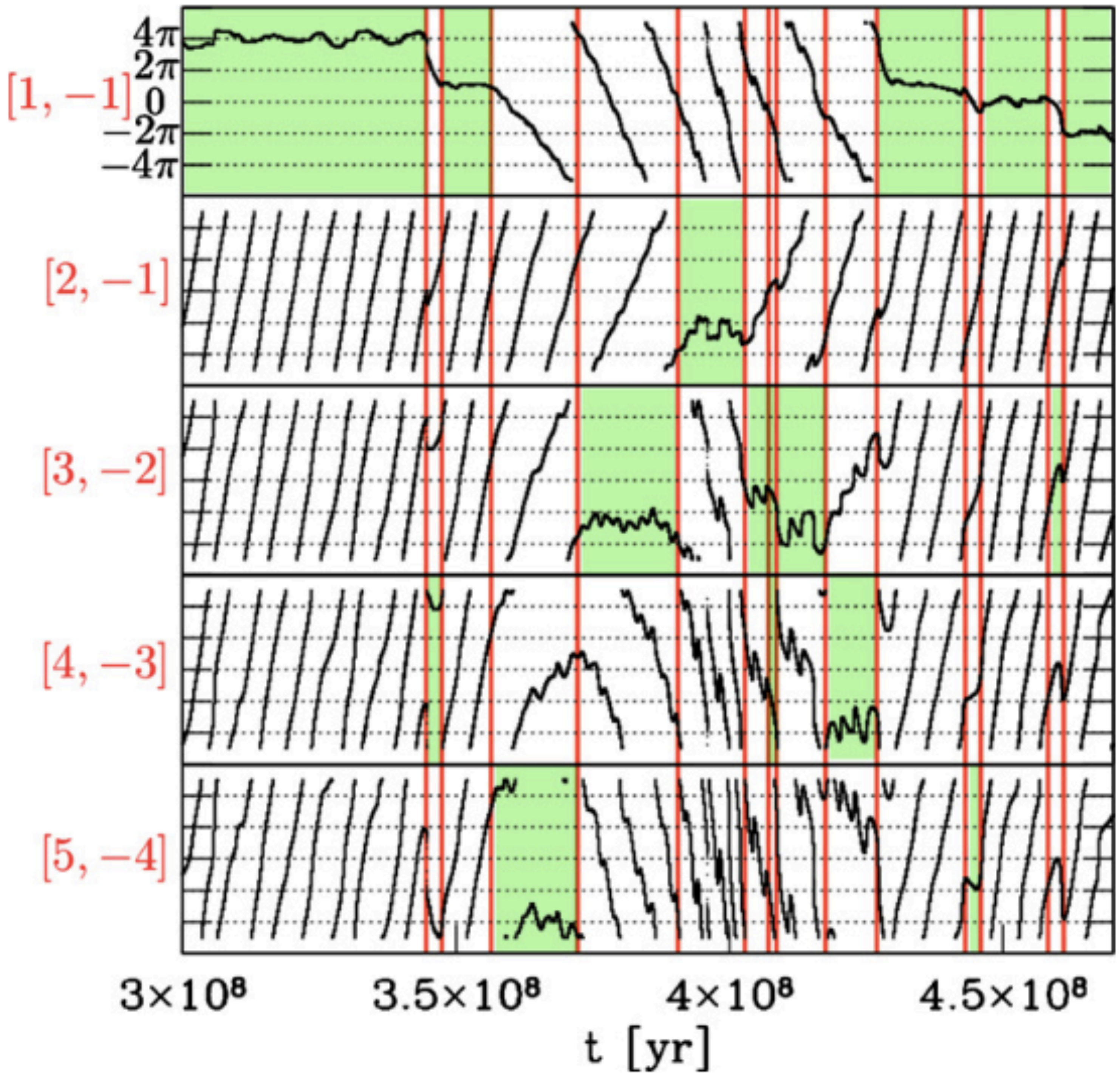
1%
with GR

60%
without GR

→ indirect confirmation of GR

Laskar & Gastineau 2009

Secular chaos



Lithwick & Wu 2011

Previous direct numerical simulations

LONGSTOP (1982)

- Outer planets only
- No instability

Digital Orrery (1988)

- Outer planets only, 800 Myr
- Pluto is chaotic

Laskar (1989)

- All planets, averaged equations
- Earth is chaotic on a 100 Myr timescale

Laskar (2009)

- All planets, full equations
- Collisions between terrestrial planets possible

Open Questions

What really drives the instability?

- Secular chaos / specific resonances

Are numerical algorithms accurate?

- So far, only 1 group was able to run such simulations.

How important are other physical effects?

- Chaotic system, small changes can lead to very distinct outcomes.

Is it theoretically possibly to prevent an instability?

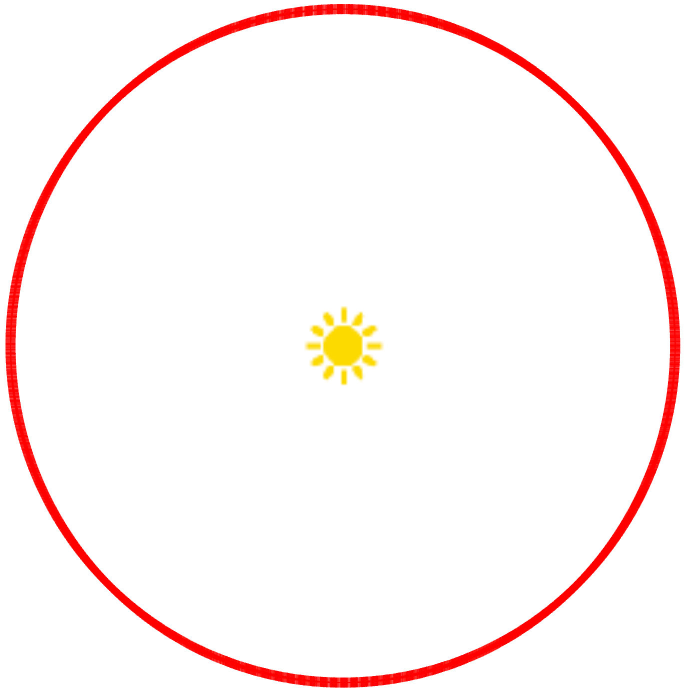
- Planetary defence on extremely long timescales.

Numerics

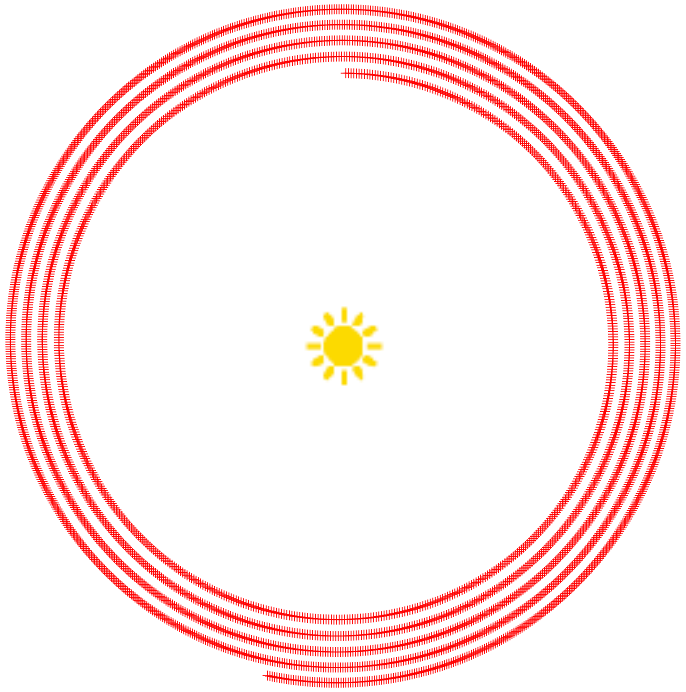
IEEE754

WHFast

Symplectic integrators



Symplectic integrator



Non-symplectic integrator

Operator Splitting

$$H = \underbrace{\frac{1}{2}p^2}_{\text{Drift}} + \underbrace{\Phi(x)}_{\text{Kick}}$$

$$x \rightarrow x + v\,\frac{\Delta t}{2}$$

$$v \rightarrow v + \nabla\Phi\,\Delta t$$

$$x \rightarrow x + v\,\frac{\Delta t}{2}$$

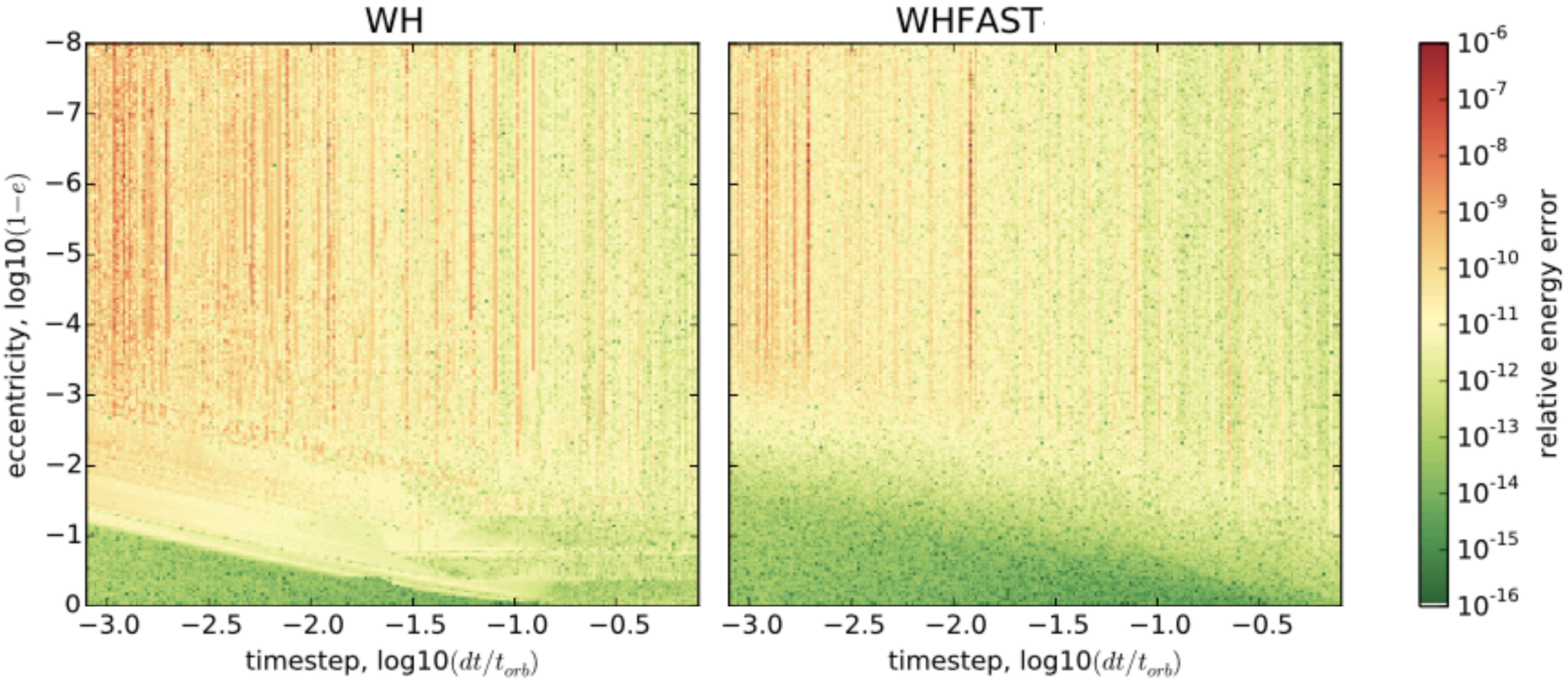
Mixed Variable Symplectic Integrator

$$H = \underset{\text{Drift}}{\boxed{\frac{1}{2}p^2 + \Phi_{\text{Sun}}(q)}} + \underset{\text{Kick}}{\boxed{\Phi_{\text{Other}}(q)}}$$

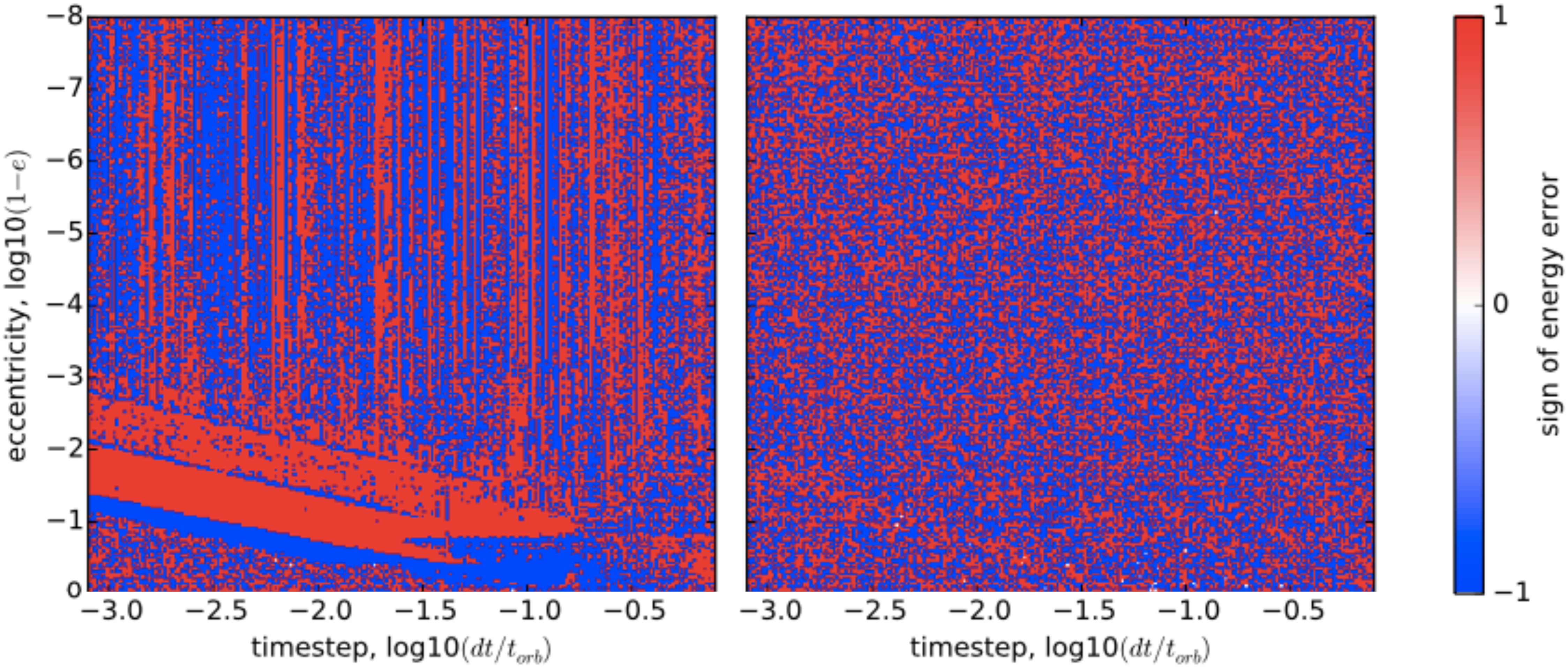
Particularly good if

$$\frac{1}{2}p^2 + \Phi_{\text{Sun}}(q) \gg \Phi_{\text{Other}}(q)$$

2-body results



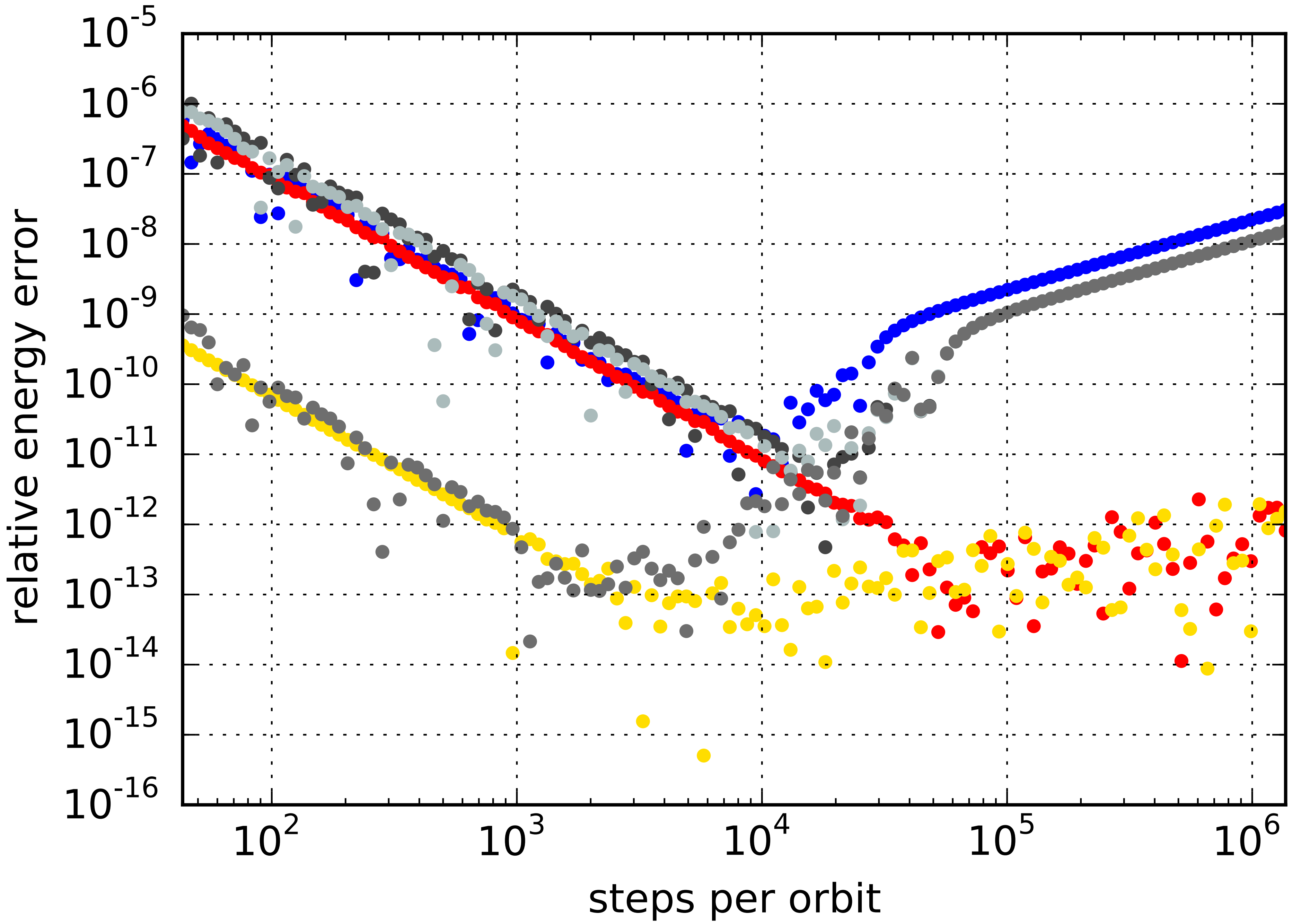
2-body results



Other optimizations

- Optimized implementation of c- and G-functions
  - Fixed number of terms in series expansion
  - Pre-calculated constants (inverse factorials)
- Fall back bisection for Newton’s method
- Optimized bias-free Jacobi-coordinate transformations
- Ordered floating point operations

Accuracy



WHFast

Mercury

Swift

Exact Reproducibility

N-body as experiments

- N-body simulations are experiments on a computer
- They do not represent the real physical system
- Simplification lead to a controllable experiment
- Yet none of the published results are reproducible

Reasons for non-reproducibility

- Source code not available
- Initial conditions not available
- Machine dependent code
    - Non standard conform code
    - Libraries
    - Non-binary file formats

Problems with non-reproducibility

- Unscientific, scientific method needs reproducible experiments
- Raises the bar for follow-up investigations
- Wasted resources, e.g. 6.2 million CPU hours by Laskar 2009

Solution

REBOUND Simulation Archive

REBOUND

Rebound is ridiculously easy to use

```
pip install rebound
```

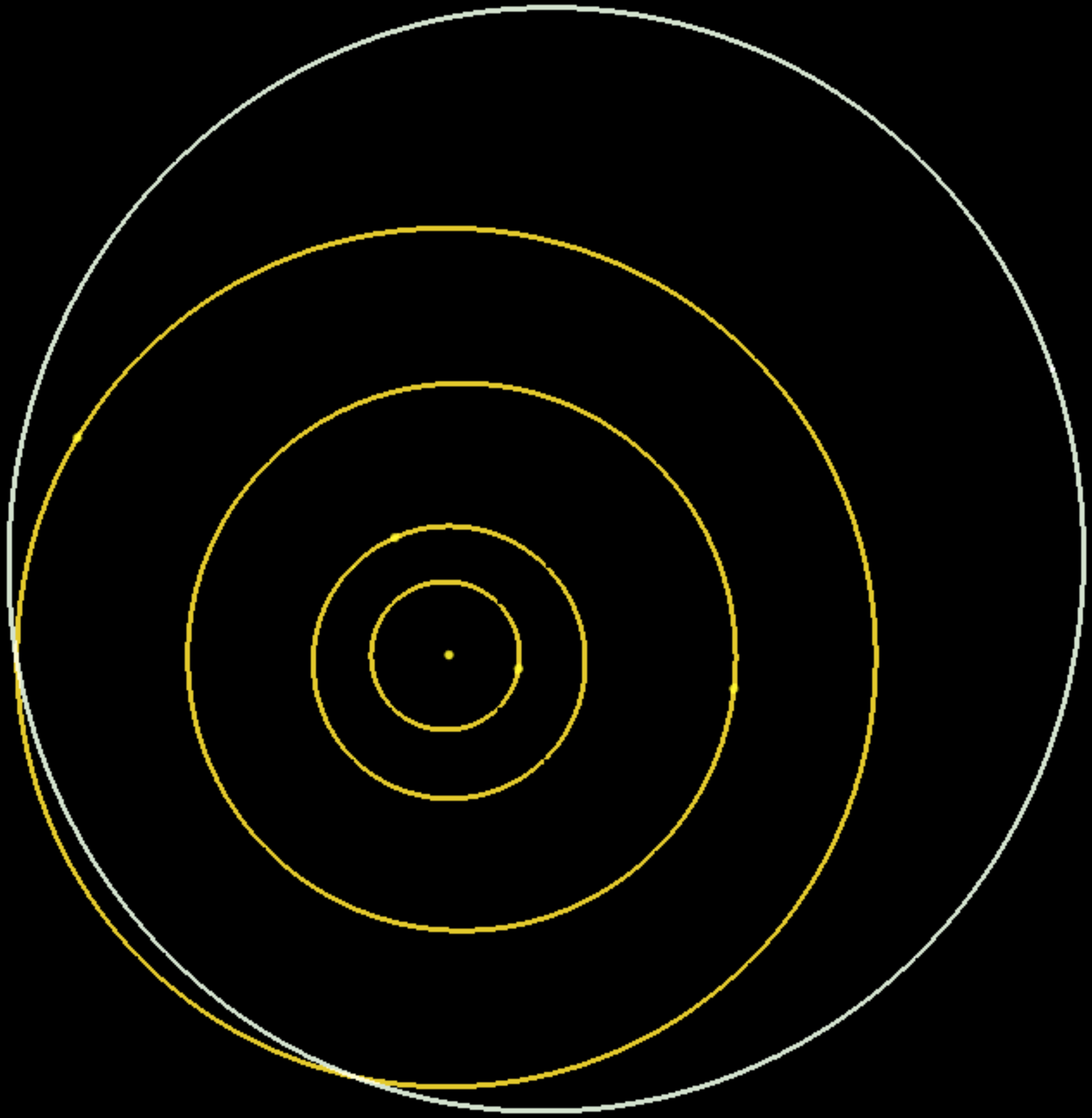
Rein and Liu (2010), Rein and Spiegel (2015), Rein and Tamayo (in prep)

Rebound is ridiculously easy to use

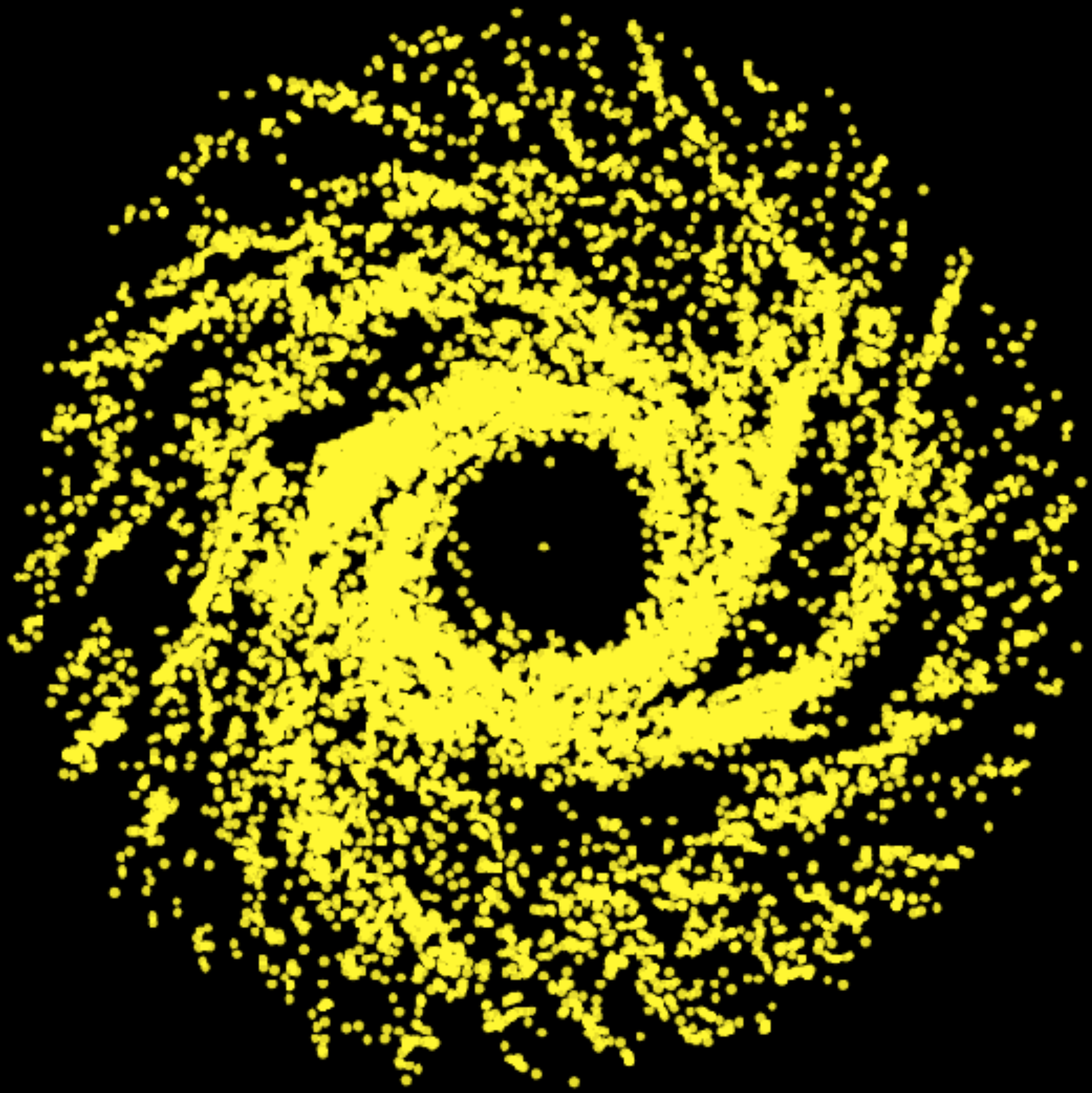
```python
import rebound

sim = rebound.Simulation()
sim.add( m=1. )
sim.add( m=1e-3, a=1. )

sim.integrate( 100. )
```

Rein and Liu (2010), Rein and Spiegel (2015), Rein and Tamayo (in prep)

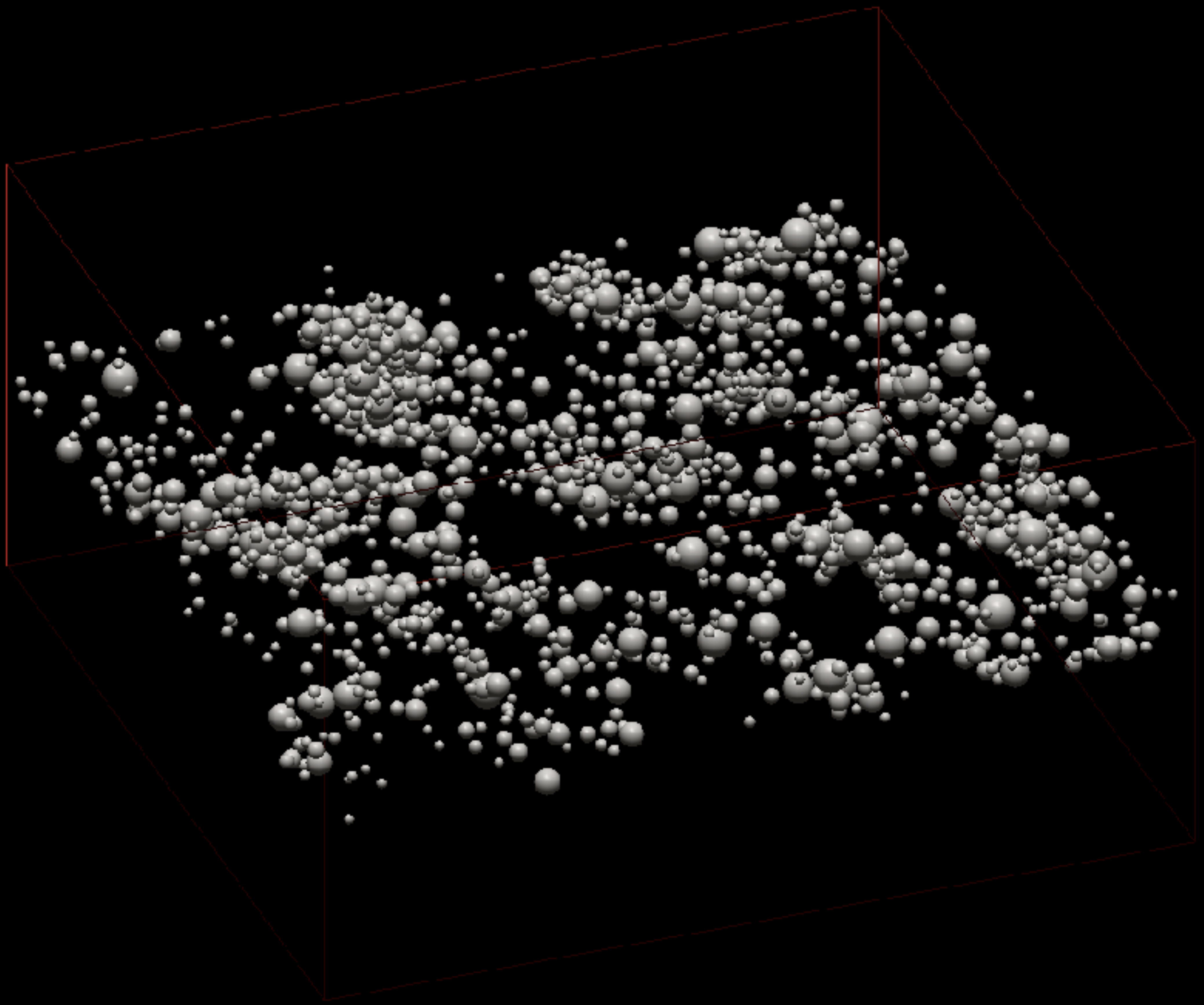
REBOUND



REBOUND



REBOUND



Demo

github.com/hannorein/rebound

Conclusions

- Integrating the Solar System for 10 Billion years is a very hard problem.
- Has only been possible in last 10 years.
- Many open questions about long term evolution of Solar System can be answered now that there is an open set of tools available.
- WHFast is an unbiased high speed symplectic integrator for planetary dynamics.
- REBOUND Simulation Archive enables reproducible experiments and a whole new paradigm when analyzing numerical simulations.