Institute for Advanced Study

Dynamical evolution of multi-planetary systems and moonlets in Saturn's Rings

Hanno Rein @ STScI September 2011

Migration in a non-turbulent disc

Multi-planetary systems

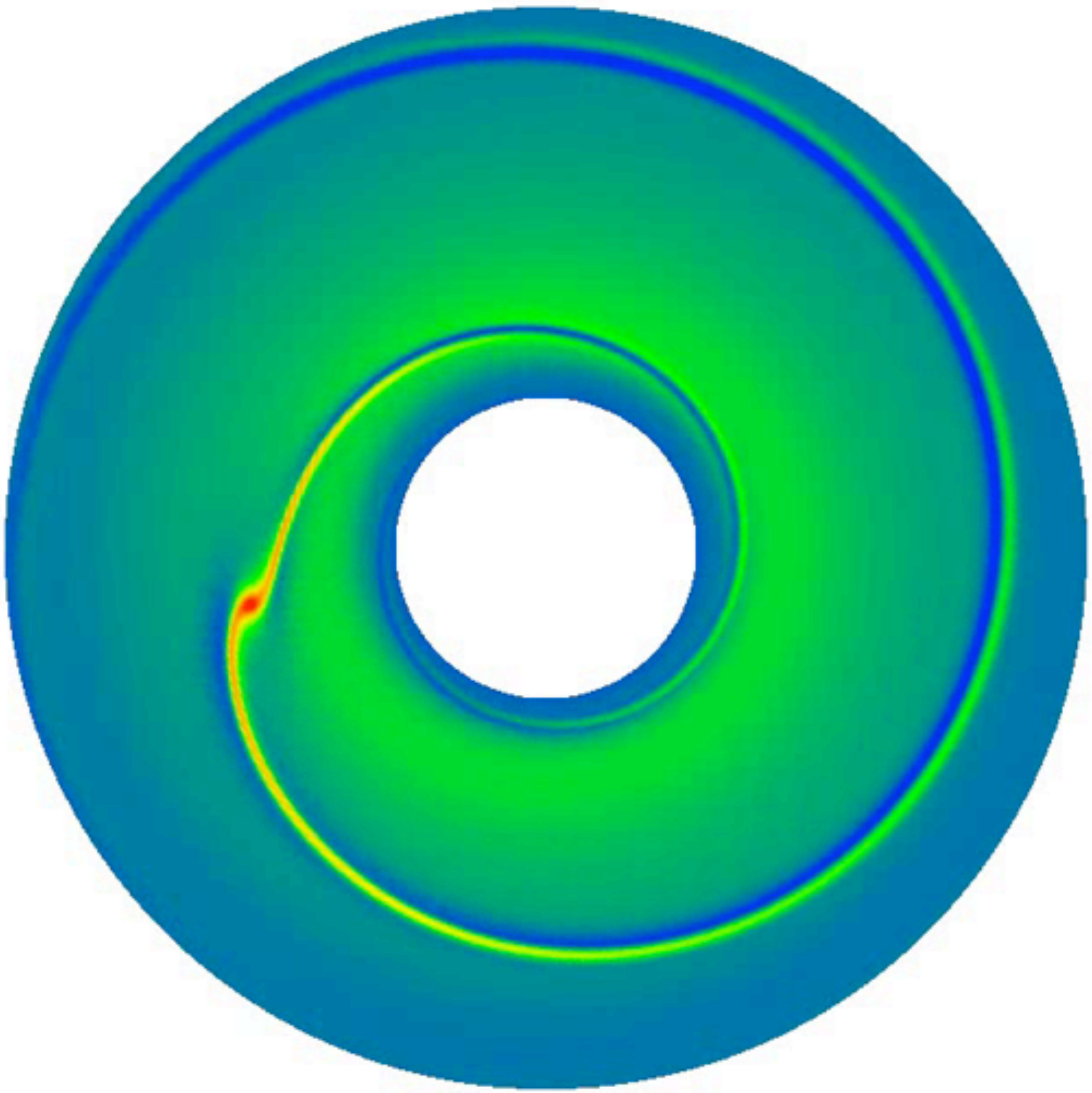
planet + disc = migration

2 planets + migration = resonance

Lee & Peale 2002, Kley & Nelson 2008, Sandor et al 2007, Rein et al 2010

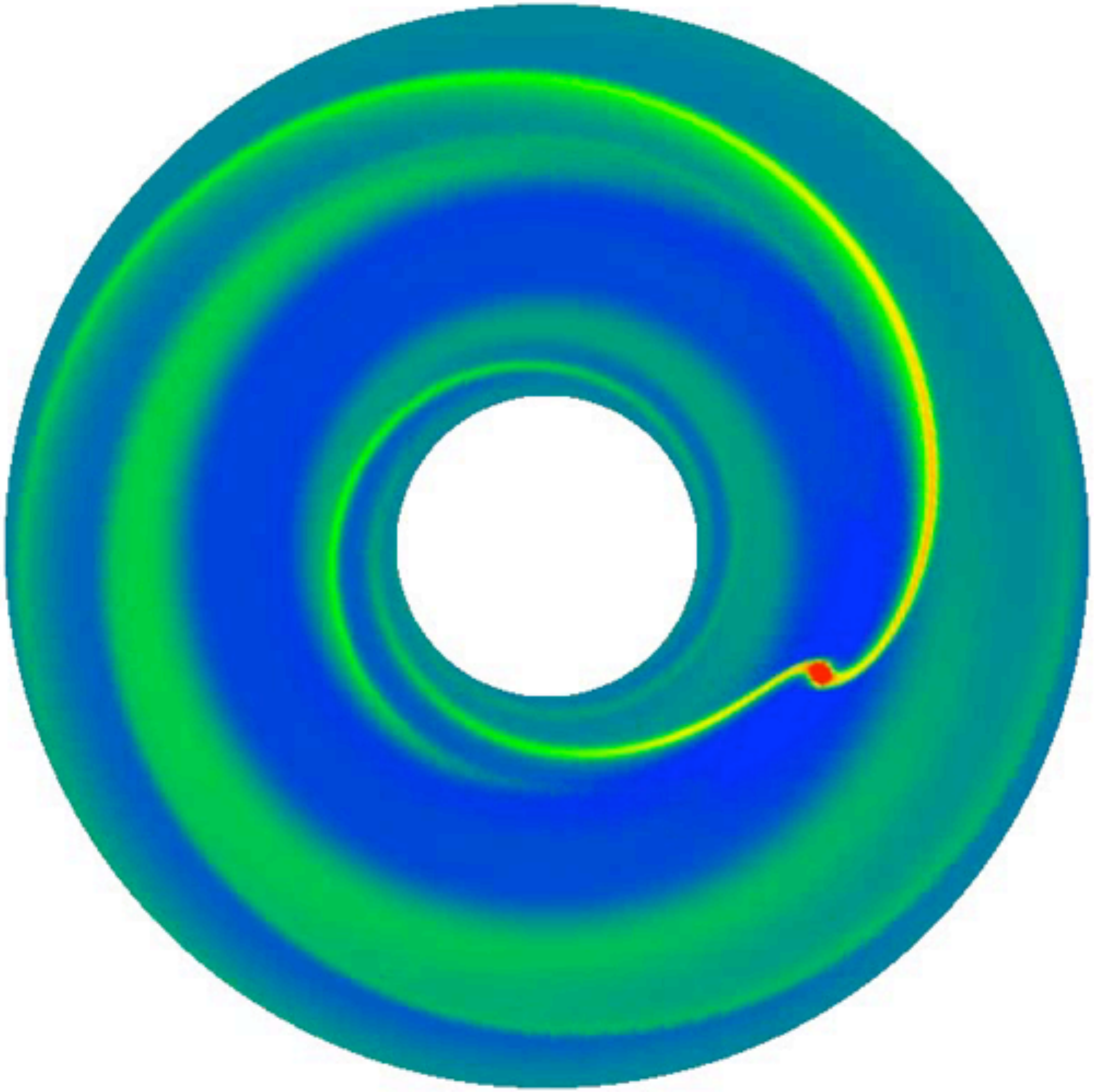
Migration - Type I

- Low mass planets
- No gap opening in disc
- Migration rate is fast
- Depends strongly on thermodynamics of the disc



Migration - Type II

- High mass planets
- Opens gap
- Follows viscous evolution of the disc



Gap opening criteria

Disc scale height

$$\frac{3}{4}\frac{H}{R_{\mathrm{Hill}}} + \frac{50 M_*}{M_p \mathcal{R}} \leq 1$$

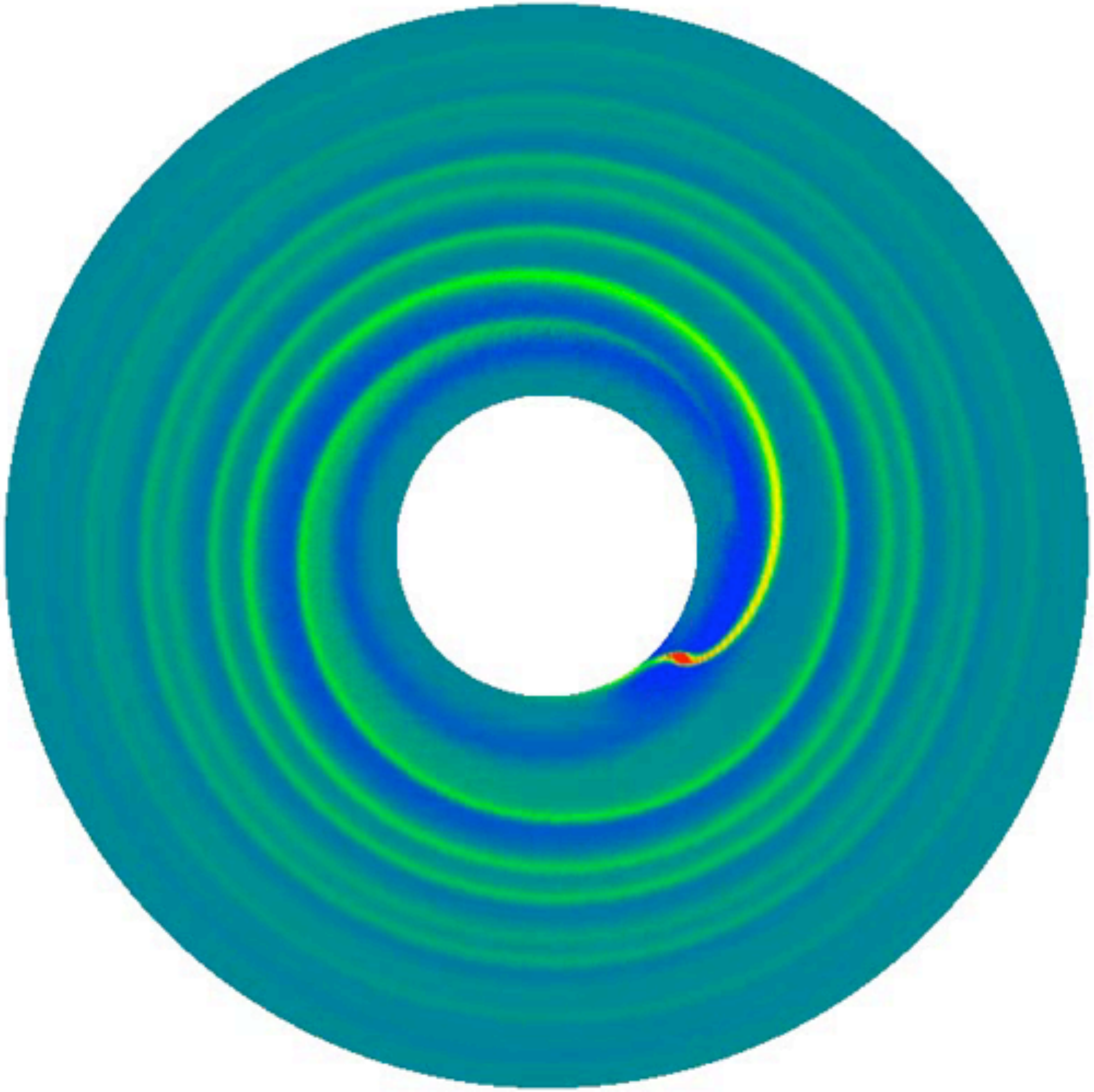
Planet mass

Viscosity

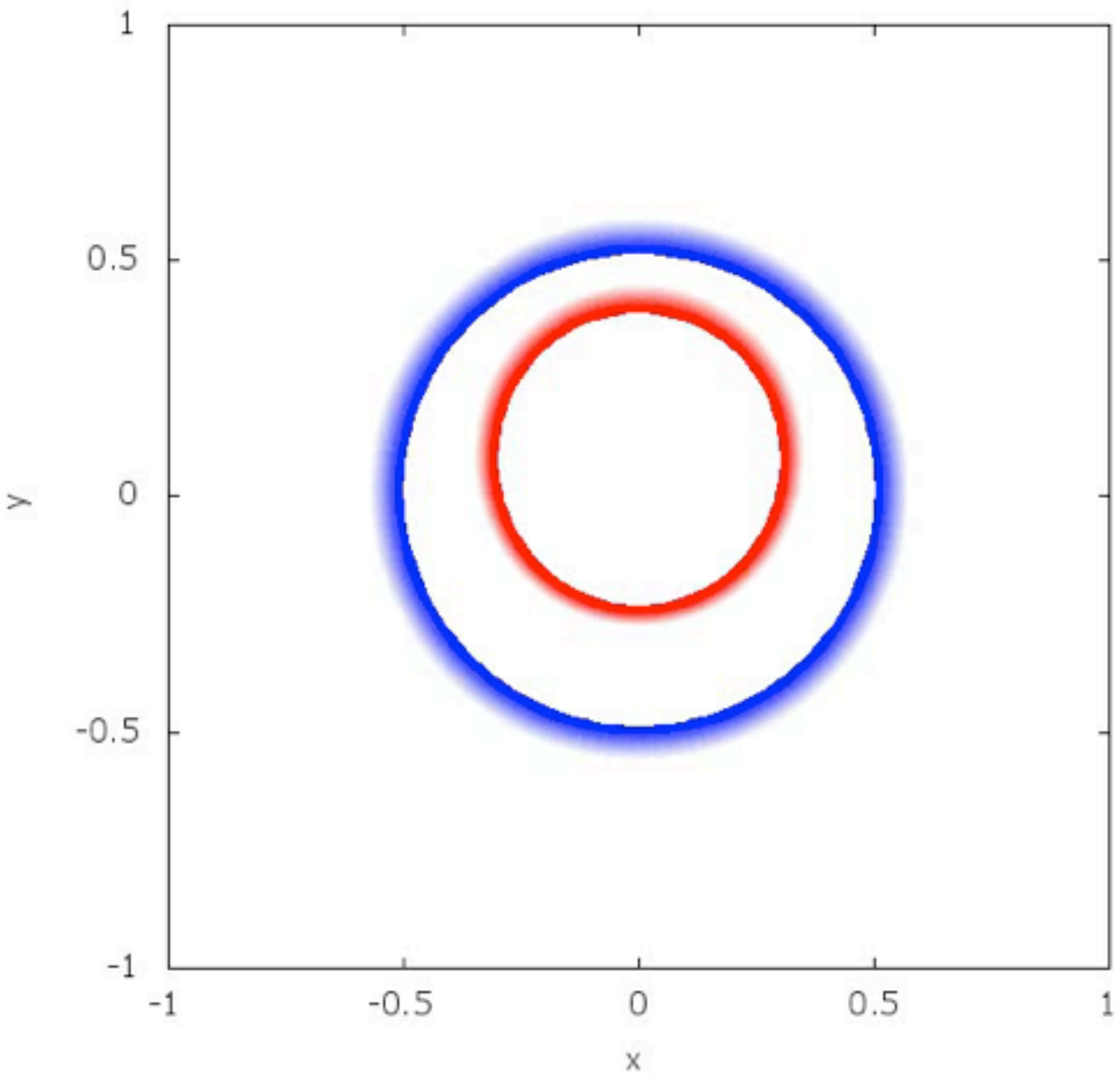
Crida et al 2006

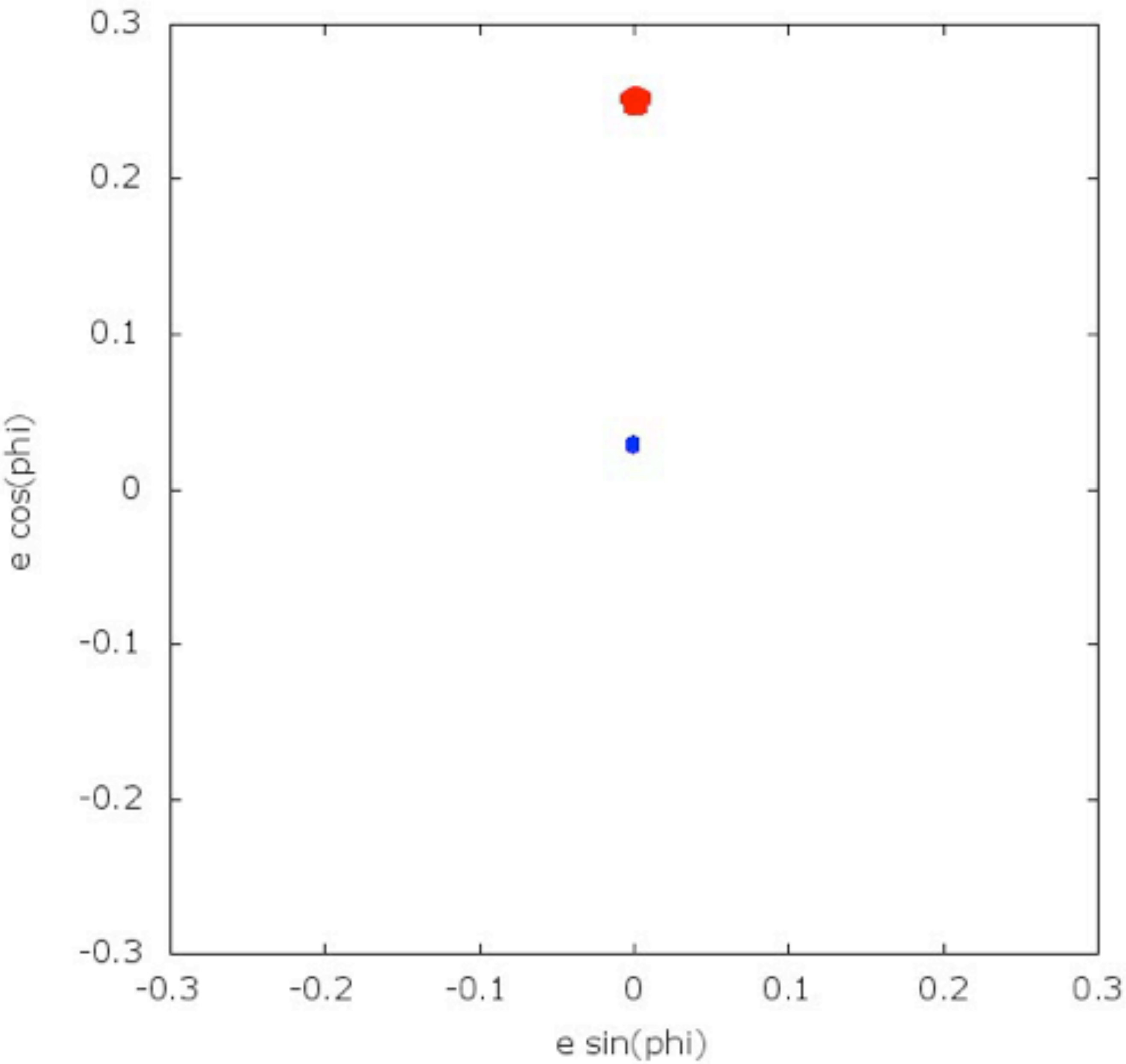
Migration - Type III

- High mass disc
- Intermediate planet mass
- Very fast



Non-turbulent resonance capture: two planets

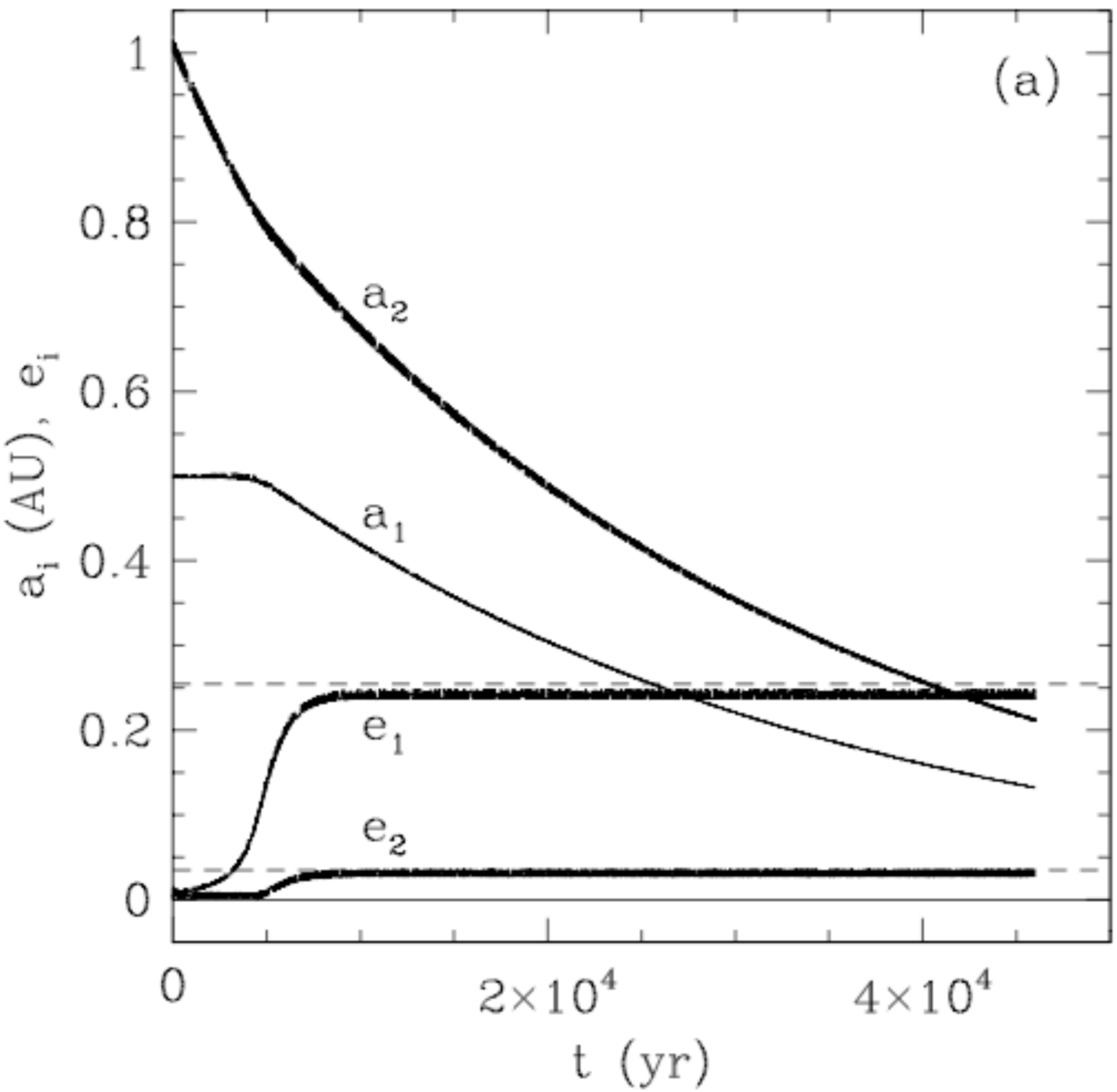




$$\phi = 2\lambda_1 - \lambda_2 - \varpi_2$$

parameters of GJ 876

GJ 876

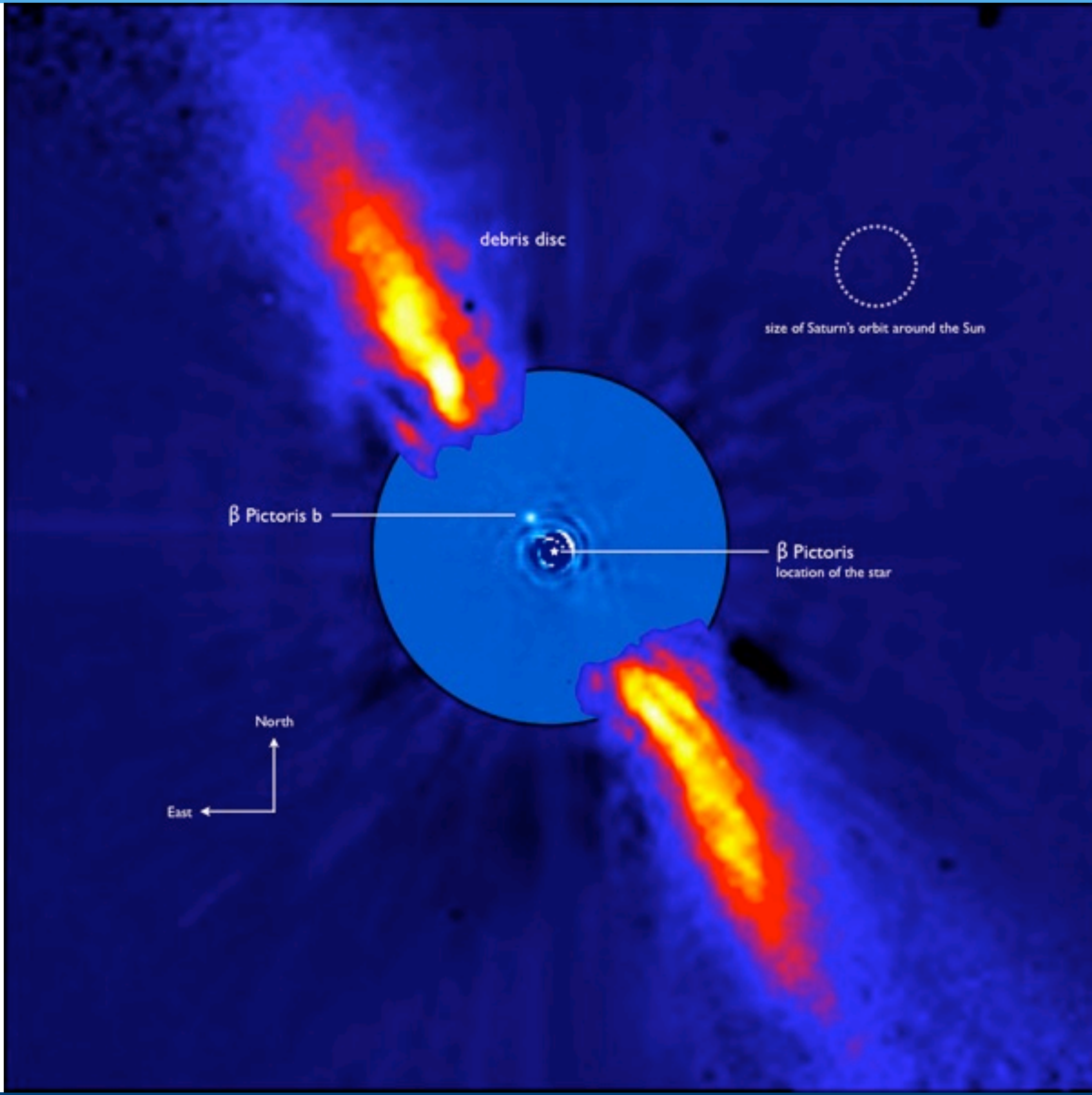


Lee & Peale 2002

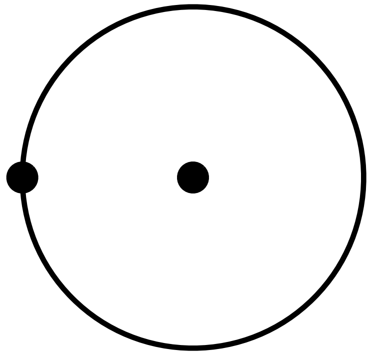
Beta Pictoris

Beta Pictoris

- Debris disc
- Nearby star (19pc)
- Planet, aligned with disc
- Asymmetries in the disc

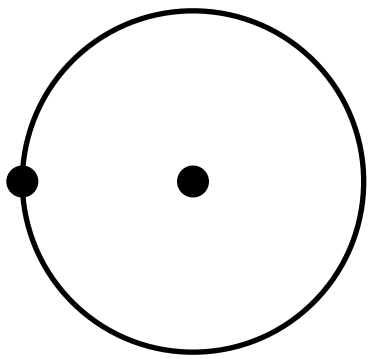


Non-turbulent resonance capture: dust

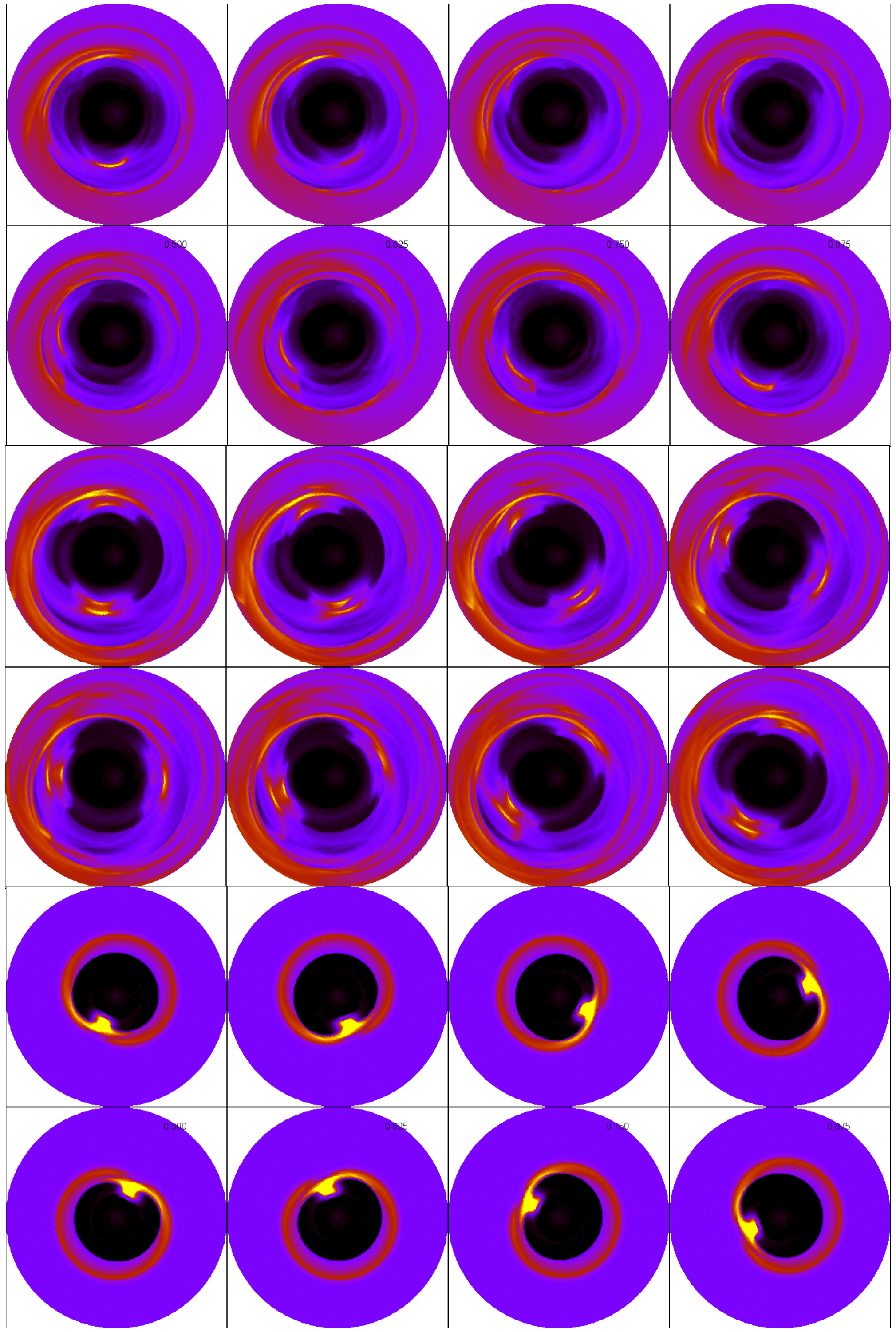


Rein & Brandeker (in preparation)

Non-turbulent resonance capture: dust

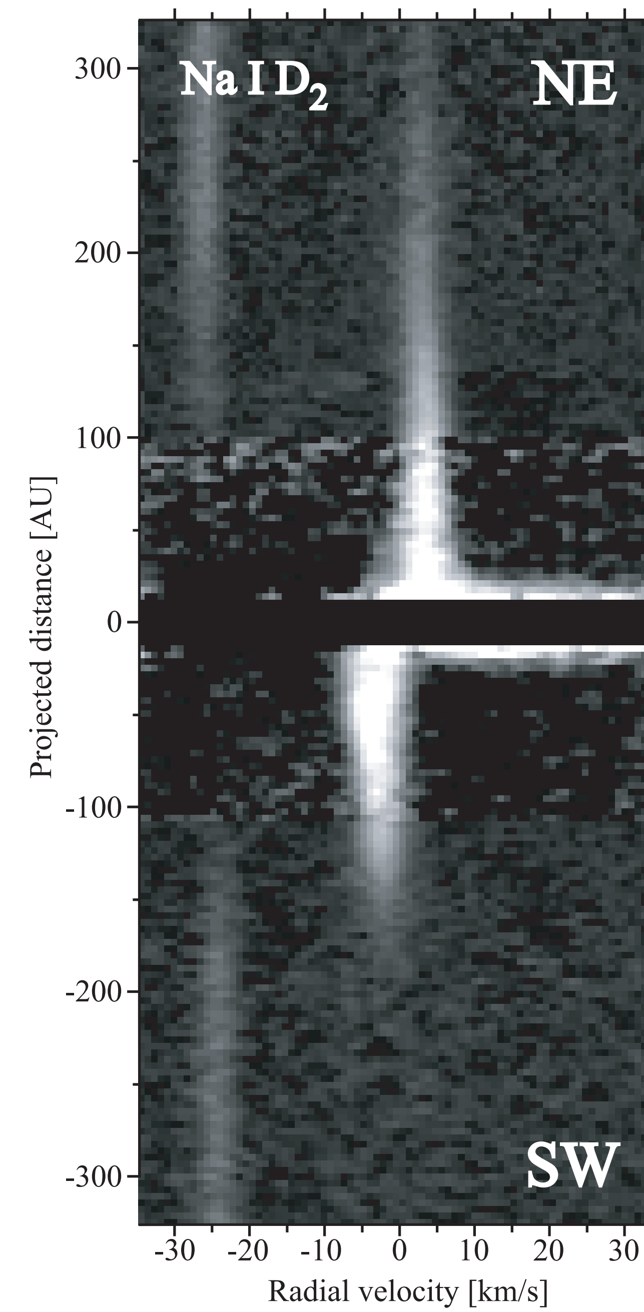


Beta Pictoris



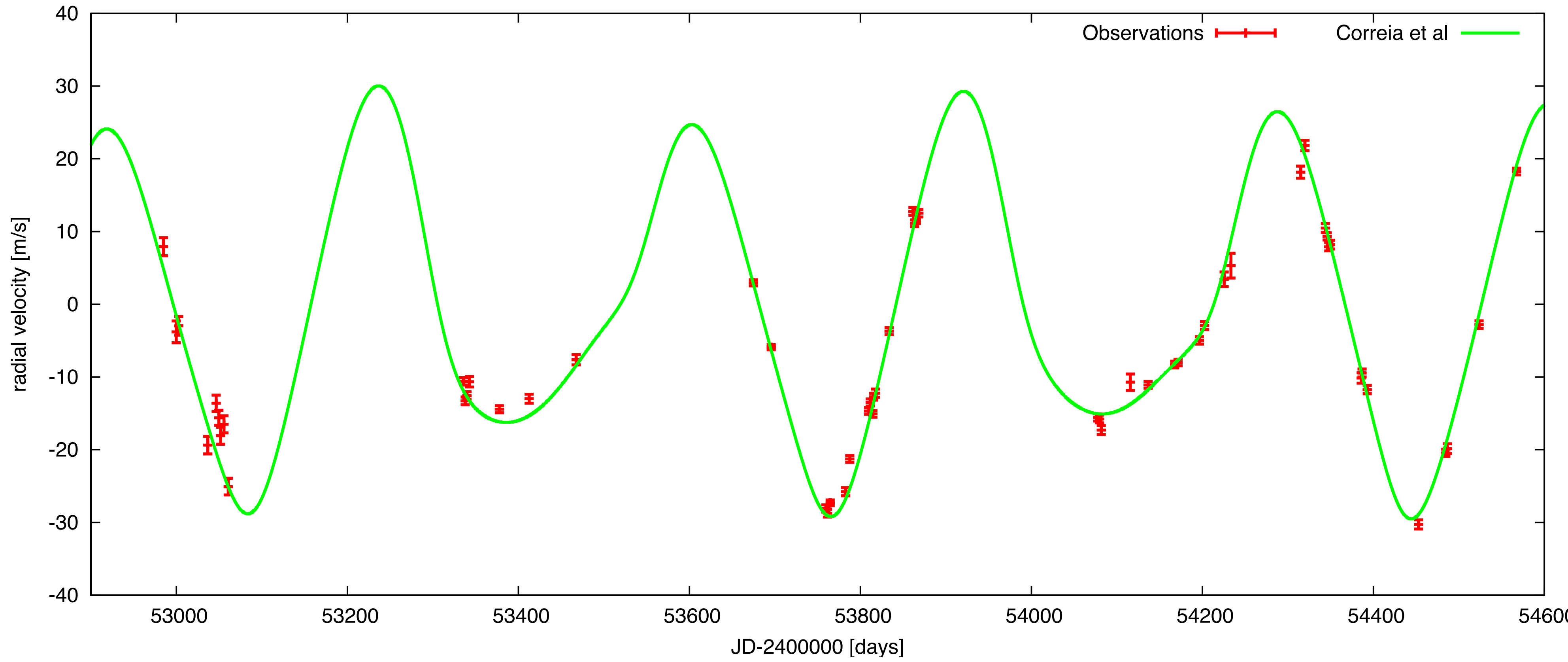
Theory

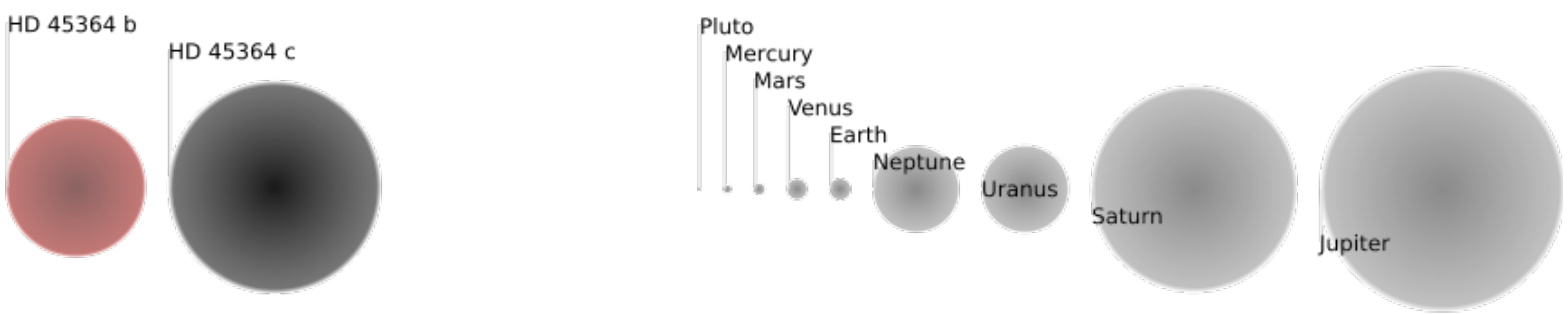
Observations



HD 45364

HD45364

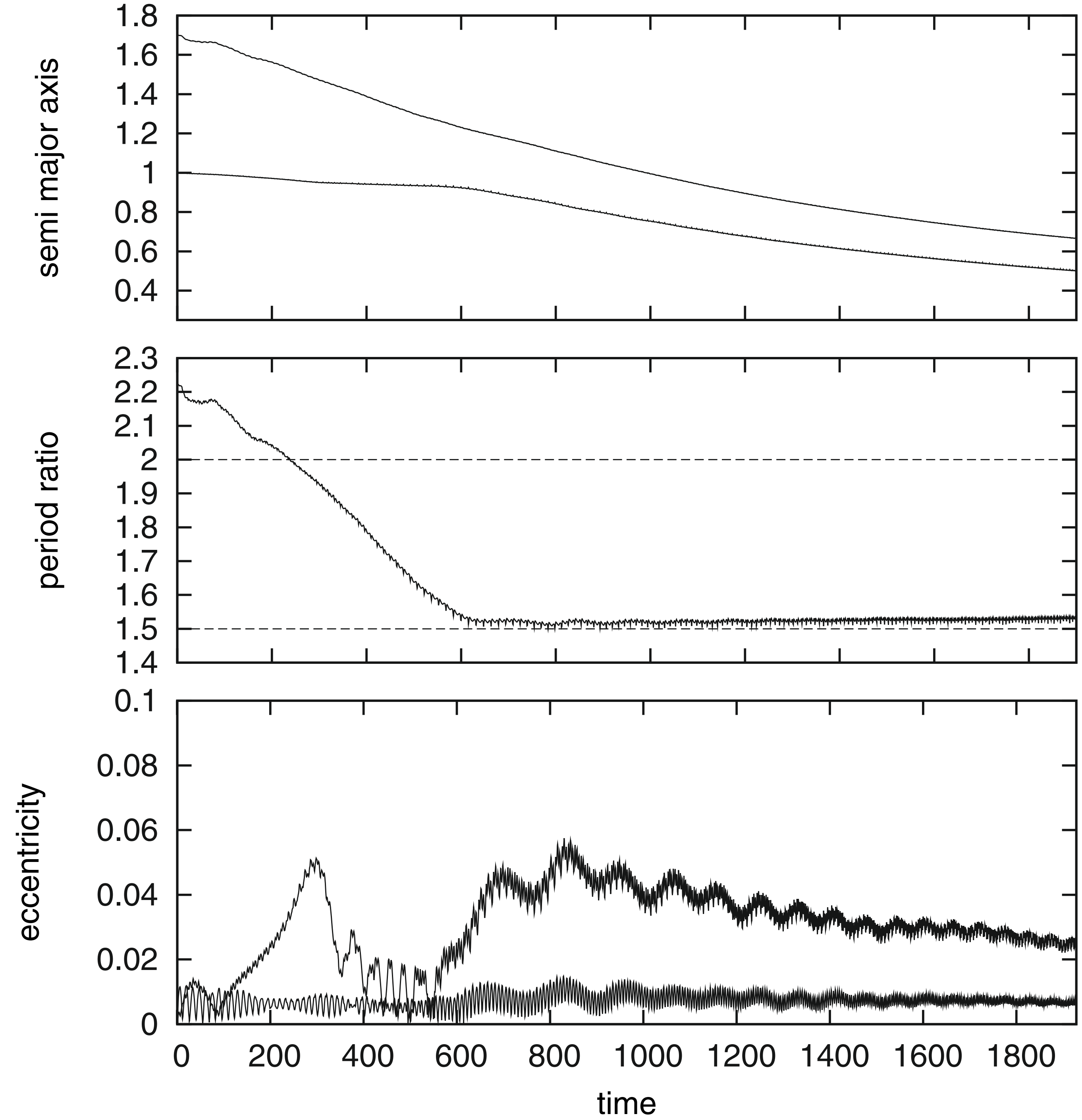




Correia et al 2009, Visual Exoplanet Catalogue

Formation scenario

- Two migrating planets
- Infinite number of resonances
- Migration speed is crucial
- Resonance width and libration period define critical migration rate



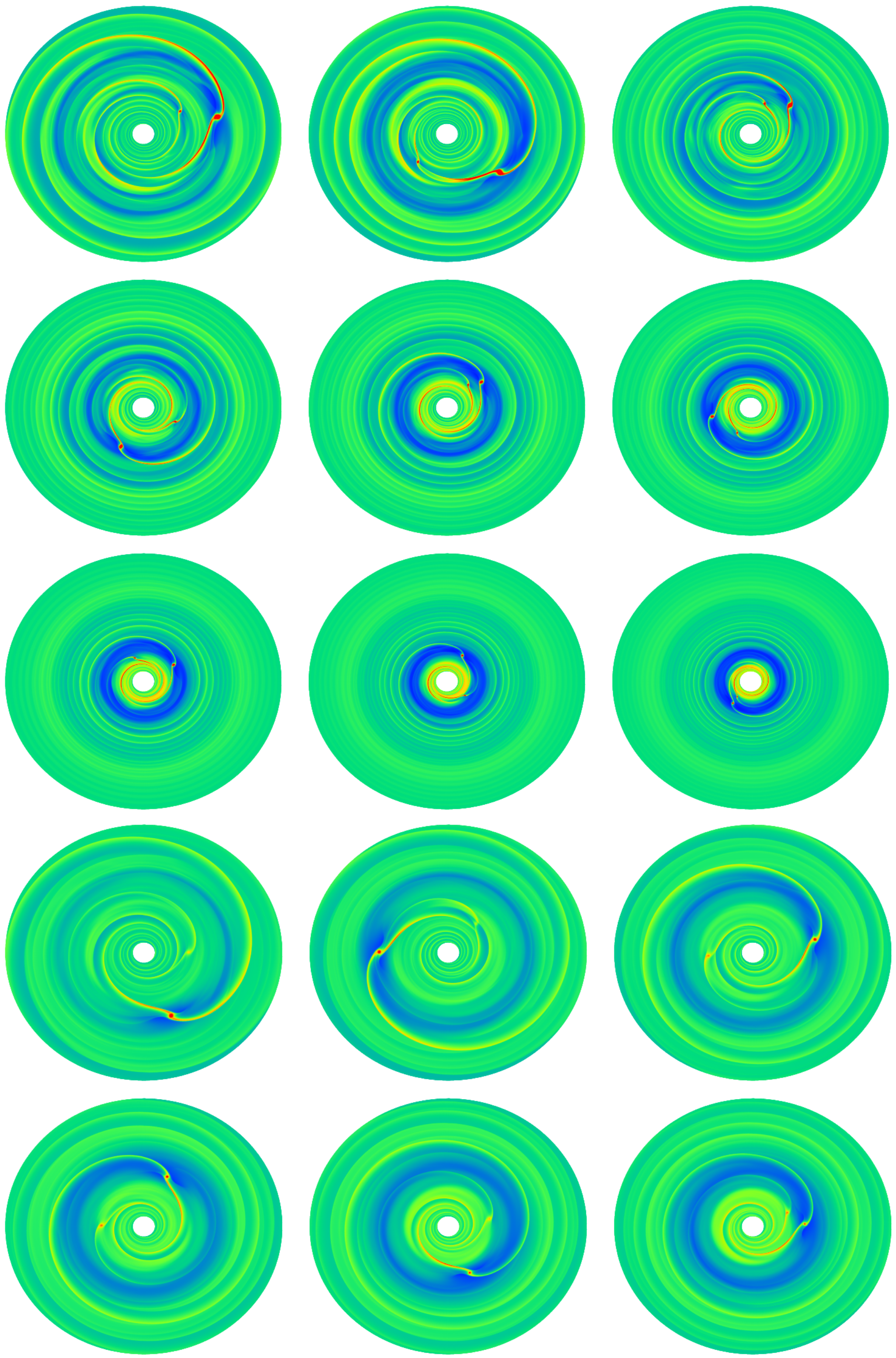
Formation scenario for HD45364

Massive disc (5 times MMSN)

- Short, rapid Type III migration
- Passage of 2:1 resonance
- Capture into 3:2 resonance

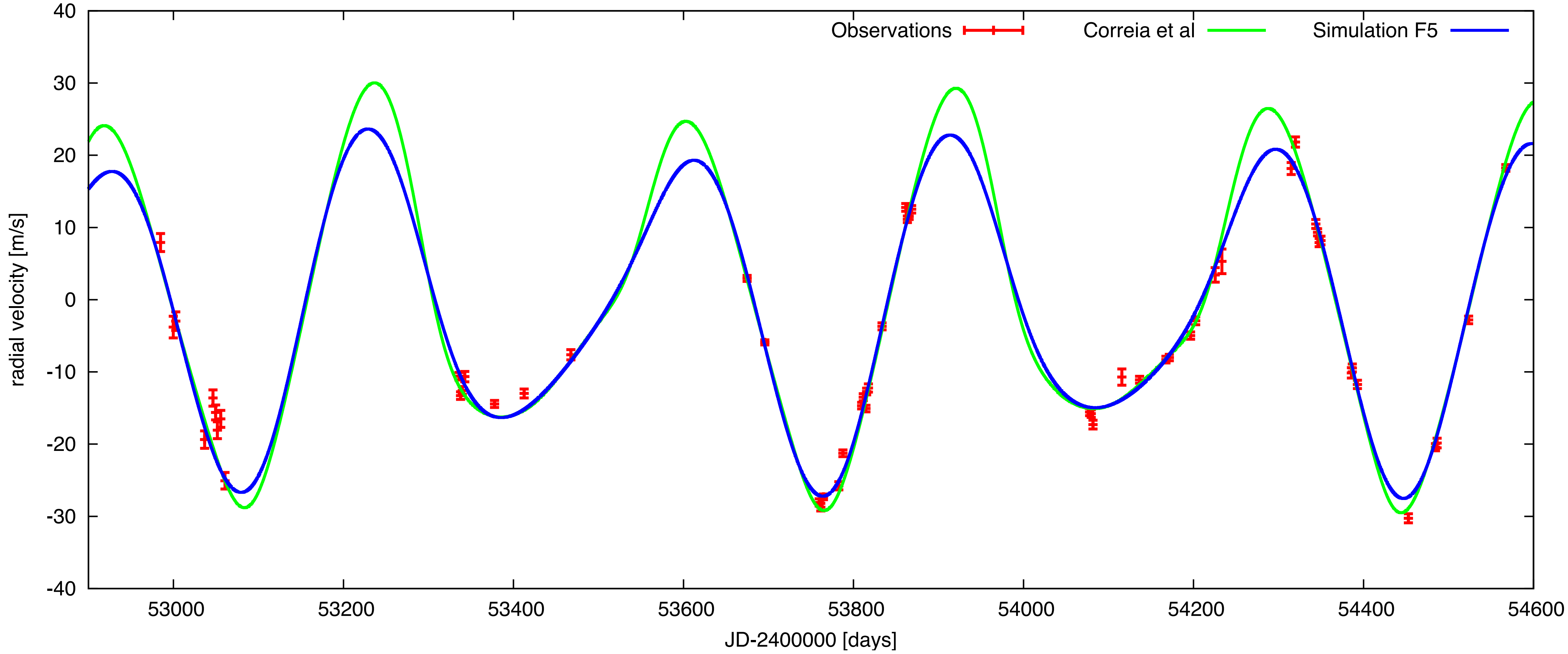
Large scale-height (0.07)

- Slow Type I migration once in resonance
- Resonance is stable
- Consistent with radiation hydrodynamics



Rein, Papaloizou & Kley 2010

Formation scenario leads to a better 'fit'



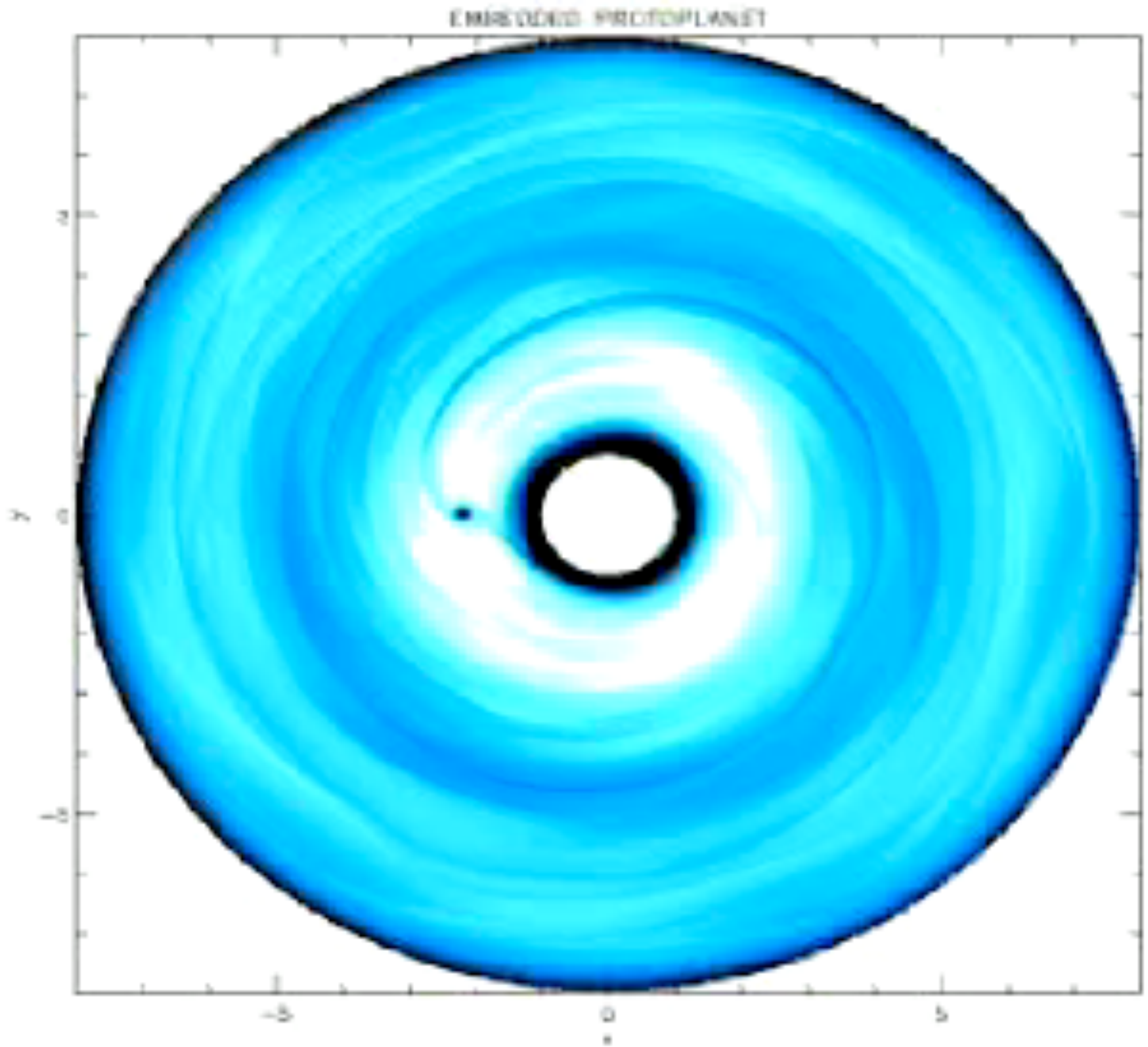
Parameter	Unit	Correia et al. (2009) b	c	Simulation **F5** b	c
$M \sin i$	$[\mathrm{M_{Jup}}]$	0.1872	0.6579	0.1872	0.6579
M_*	$[M_\odot]$	0.82		0.82	
a	[AU]	0.6813	0.8972	0.6804	0.8994
e		0.17 ± 0.02	0.097 ± 0.012	0.036	0.017
λ	[deg]	105.8 ± 1.4	269.5 ± 0.6	352.5	153.9
ϖ^a	[deg]	162.6 ± 6.3	7.4 ± 4.3	87.9	292.2
$\sqrt{\chi^2}$		2.79		2.76^b (3.51)	
Date	[JD]	2453500		2453500	

Rein, Papaloizou & Kley 2010

Migration in a turbulent disc

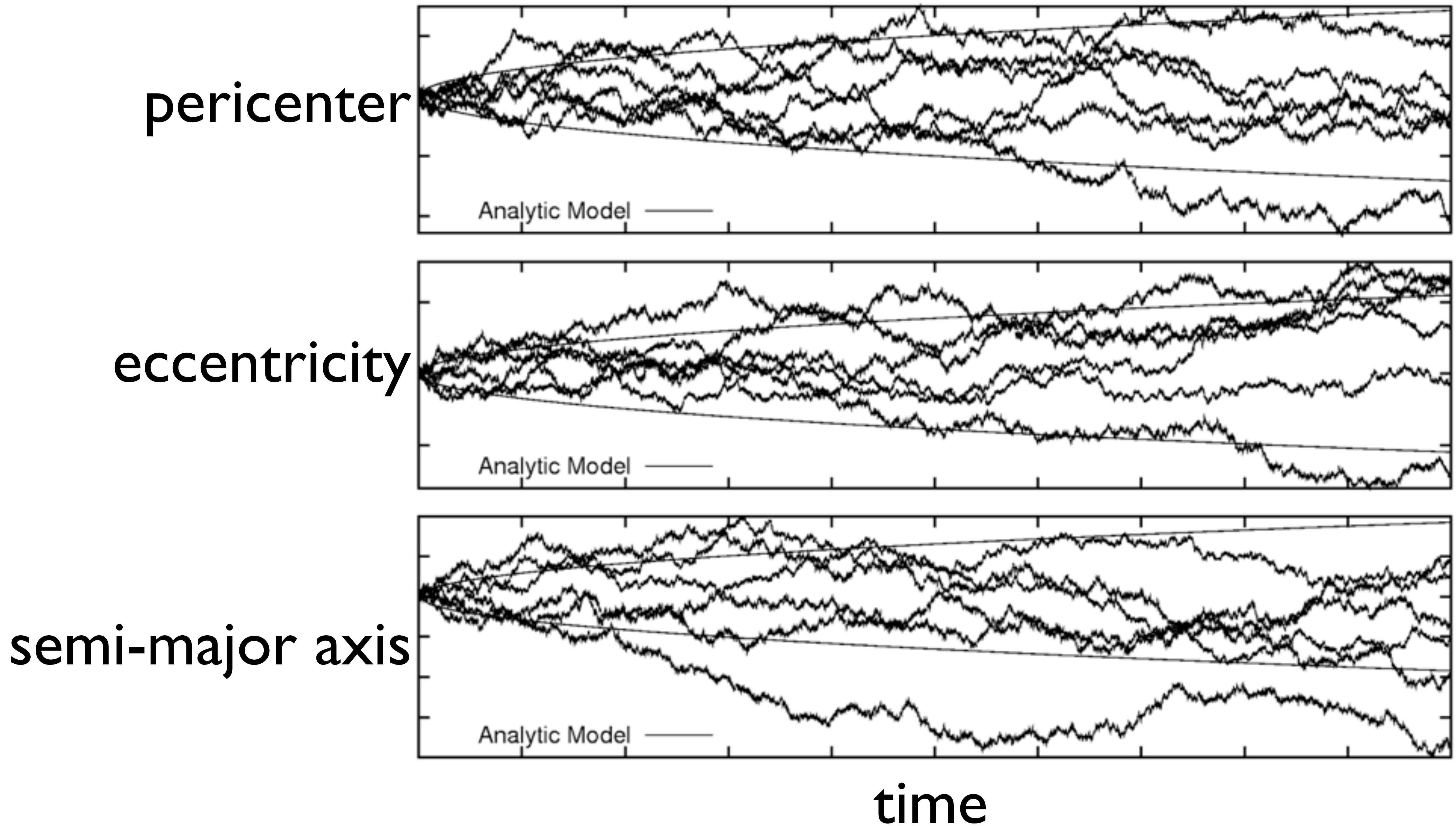
Turbulent disc

- Angular momentum transport
- Magnetorotational instability (MRI)
- Density perturbations interact gravitationally with planets
- Stochastic forces lead to random walk
- Large uncertainties in strength of forces



Animation from Nelson & Papaloizou 2004
Random forces measured by Laughlin et al. 2004, Nelson 2005, Oischi et al. 2007

Random walk



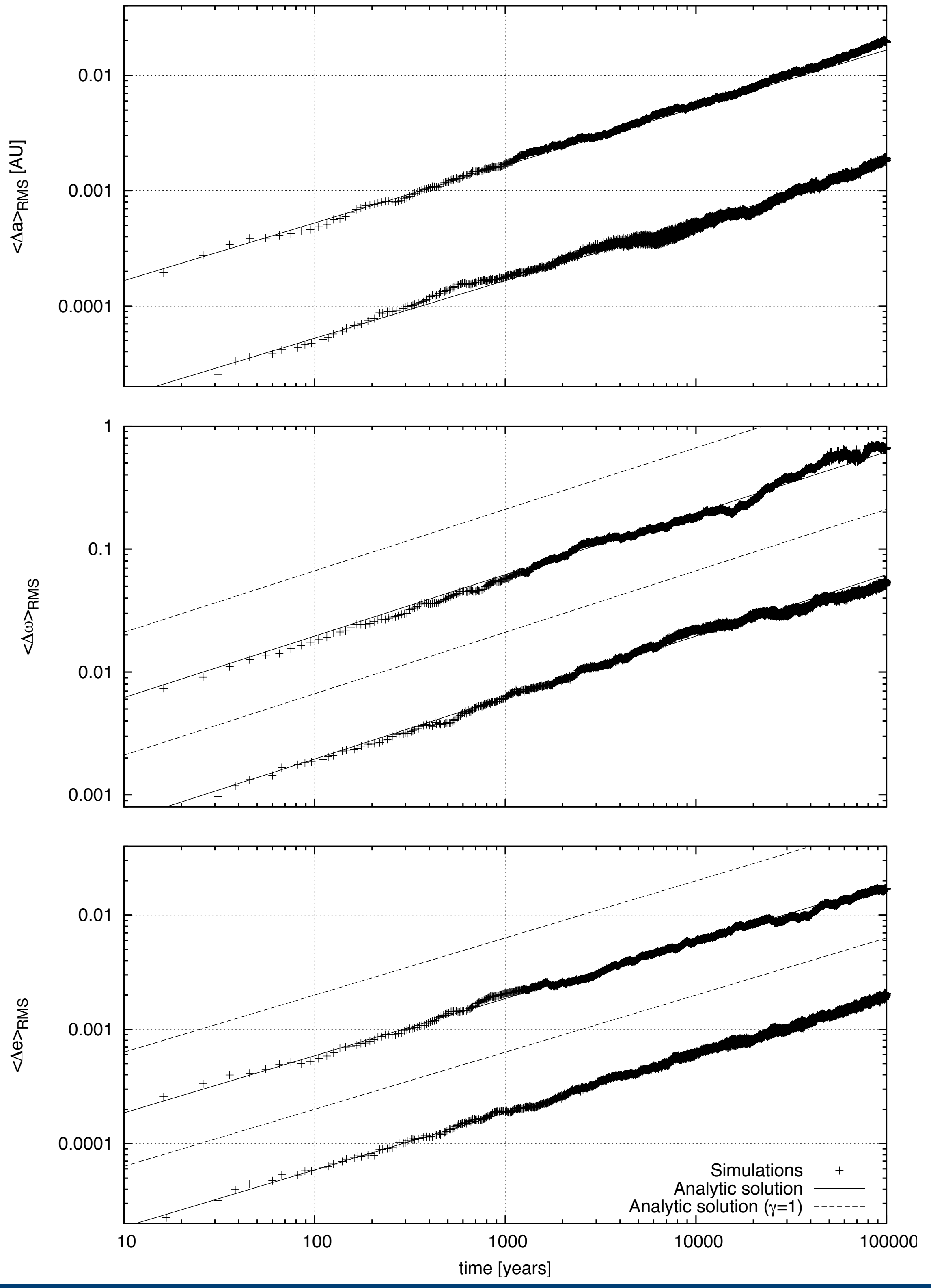
pericenter
eccentricity
semi-major axis
Analytic Model
Analytic Model
Analytic Model
time

Correction factors are important

$$(\Delta a)^2 = 4\frac{Dt}{n^2}$$

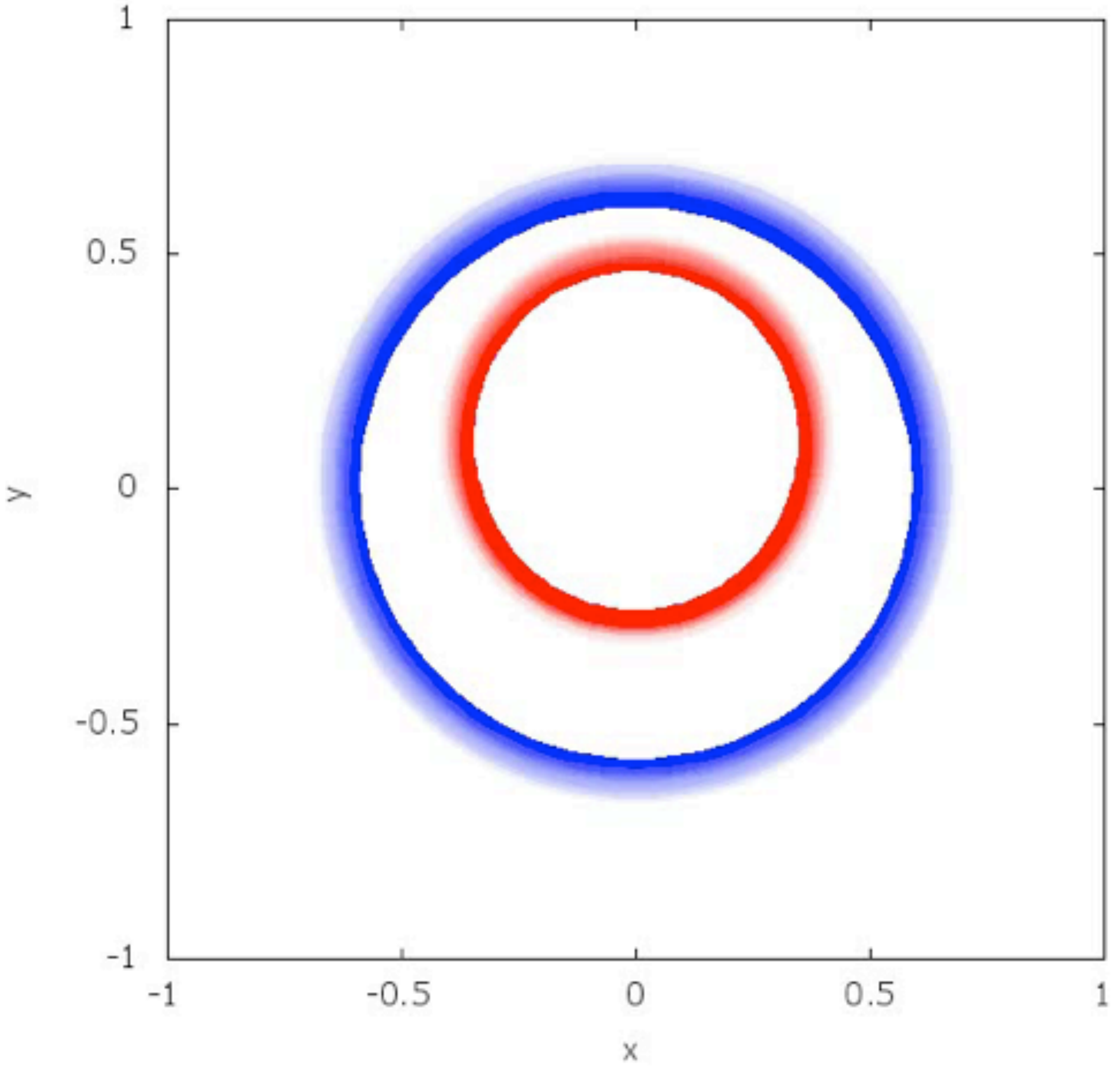
$$(\Delta \varpi)^2 = \frac{2.5}{e^2}\frac{\gamma D t}{n^2 a^2}$$

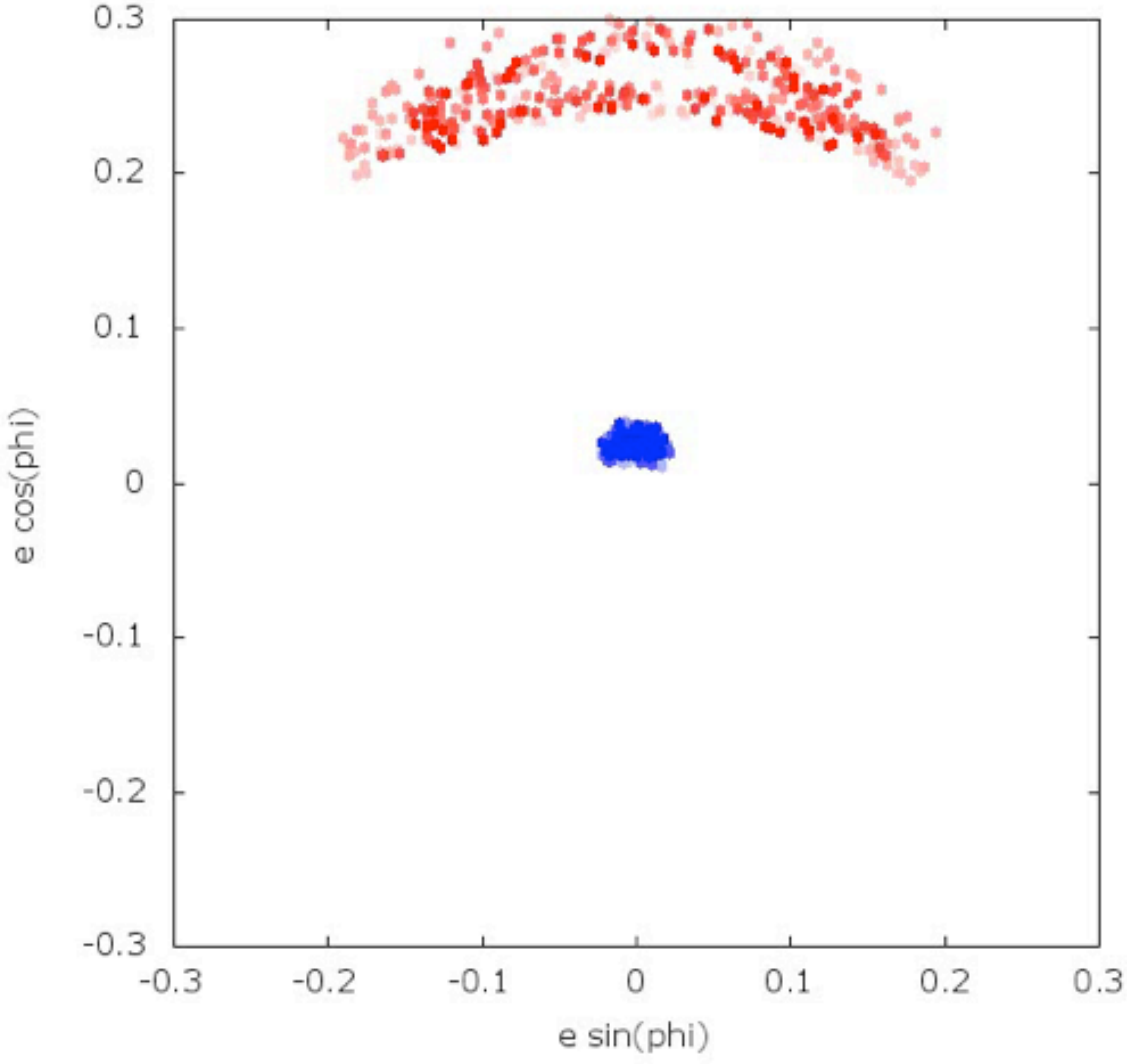
$$(\Delta e)^2 = 2.5\frac{\gamma D t}{n^2 a^2}$$



Rein & Papaloizou 2009, Adams et al 2009, Rein 2010

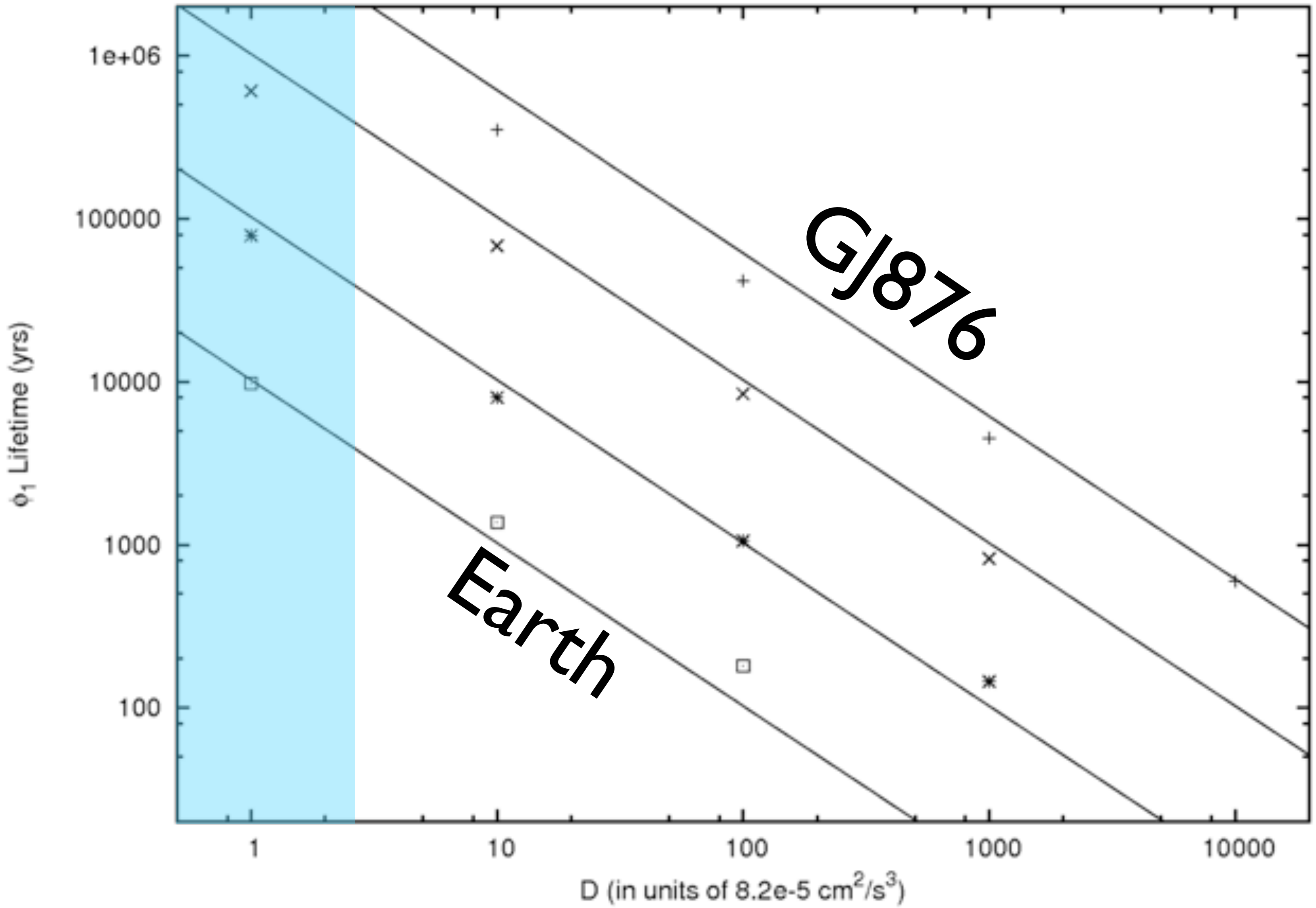
Two planets: turbulent resonance capture





Rein & Papaloizou 2009

Multi-planetary systems in mean motion resonance

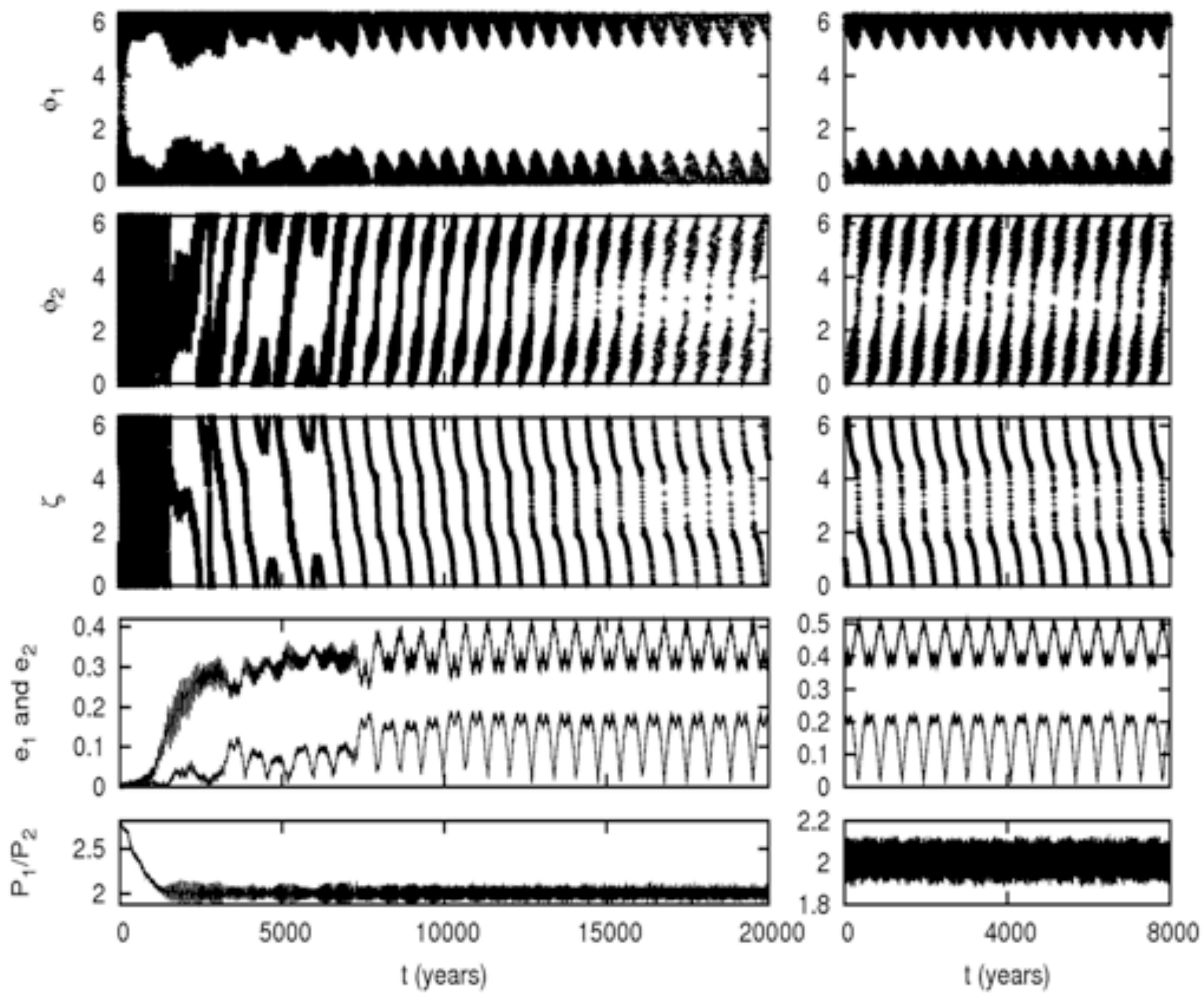


- Stability of multi-planetary systems depends strongly on diffusion coefficient
- Most planetary systems are stable for entire disc lifetime

but

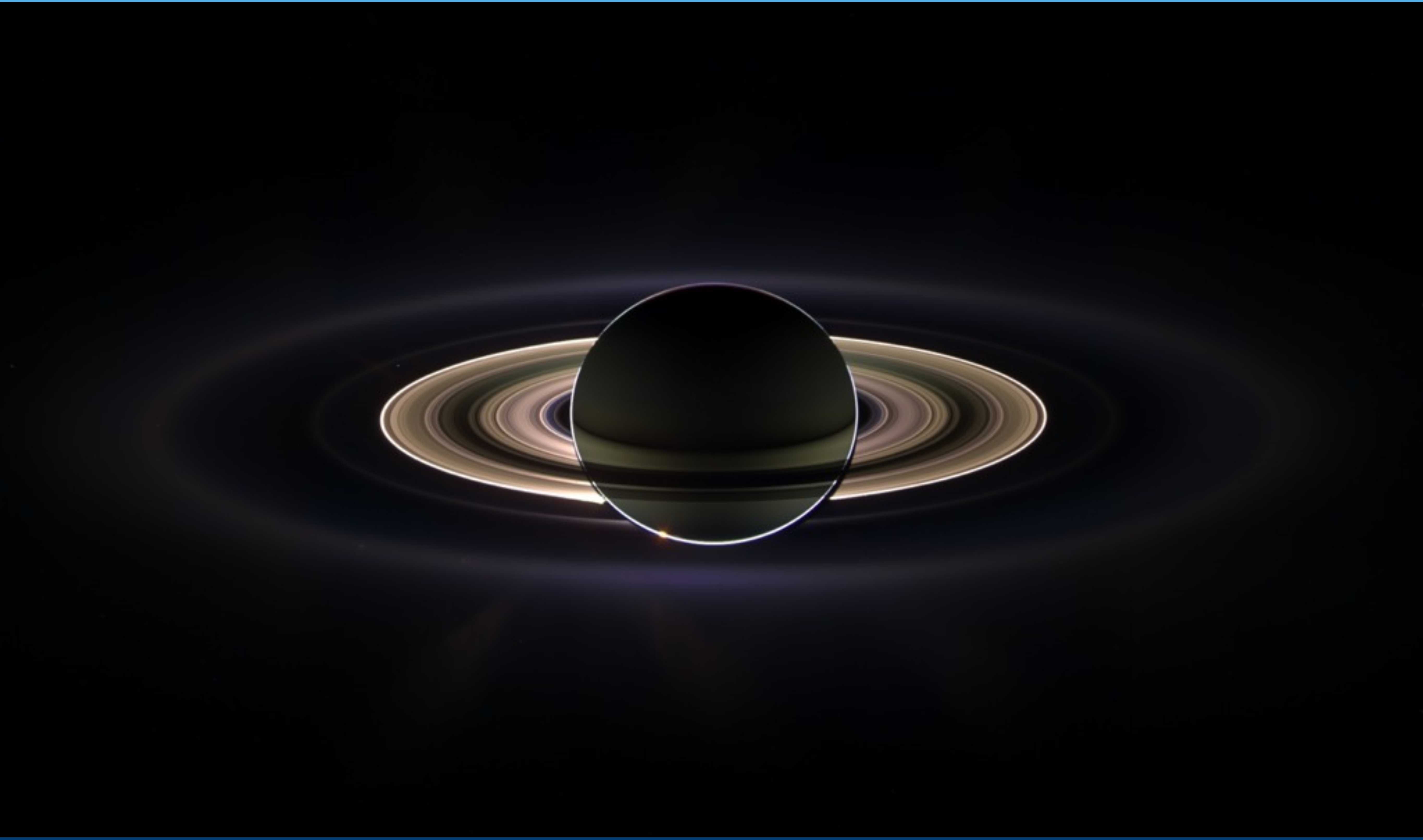
Modification of libration patterns

- HD128311 has a very peculiar libration pattern
- Can not be reproduced by convergent migration alone
- Turbulence can explain it
- More multi-planetary systems needed for statistical argument



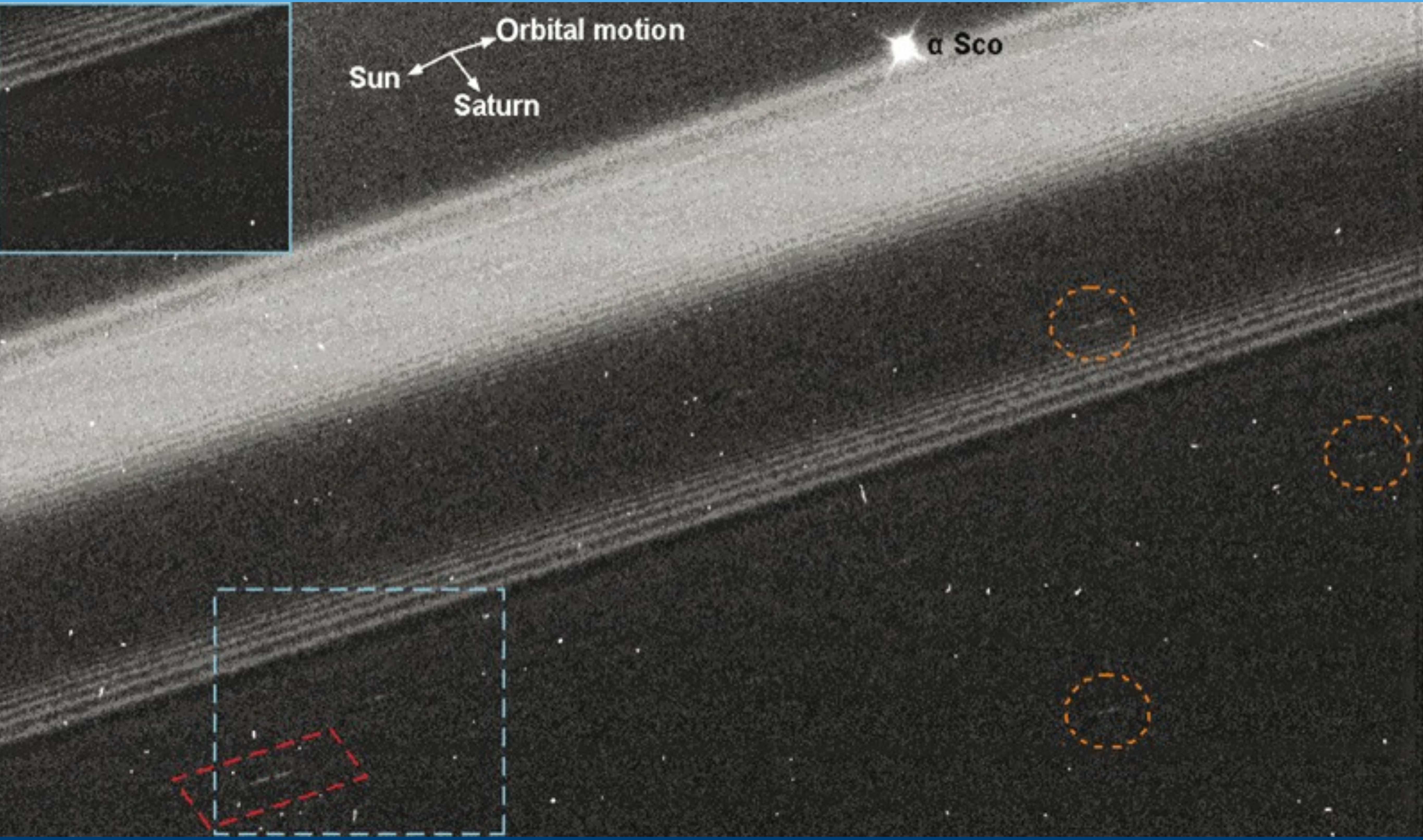
Moonlets in Saturn's Rings
I. Observations

Cassini spacecraft



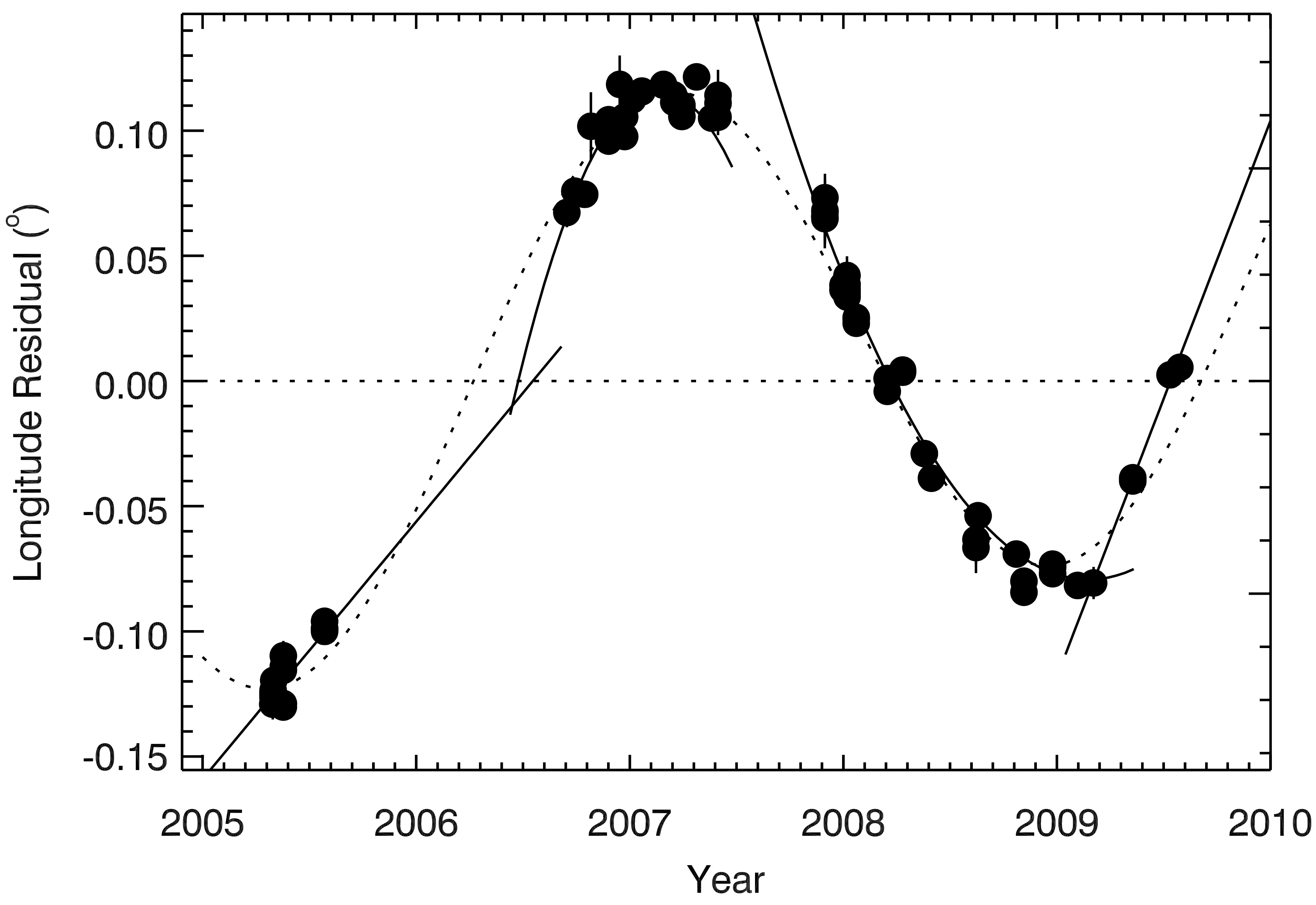
NASA/JPL/Space Science Institute

Propeller structures in A-ring



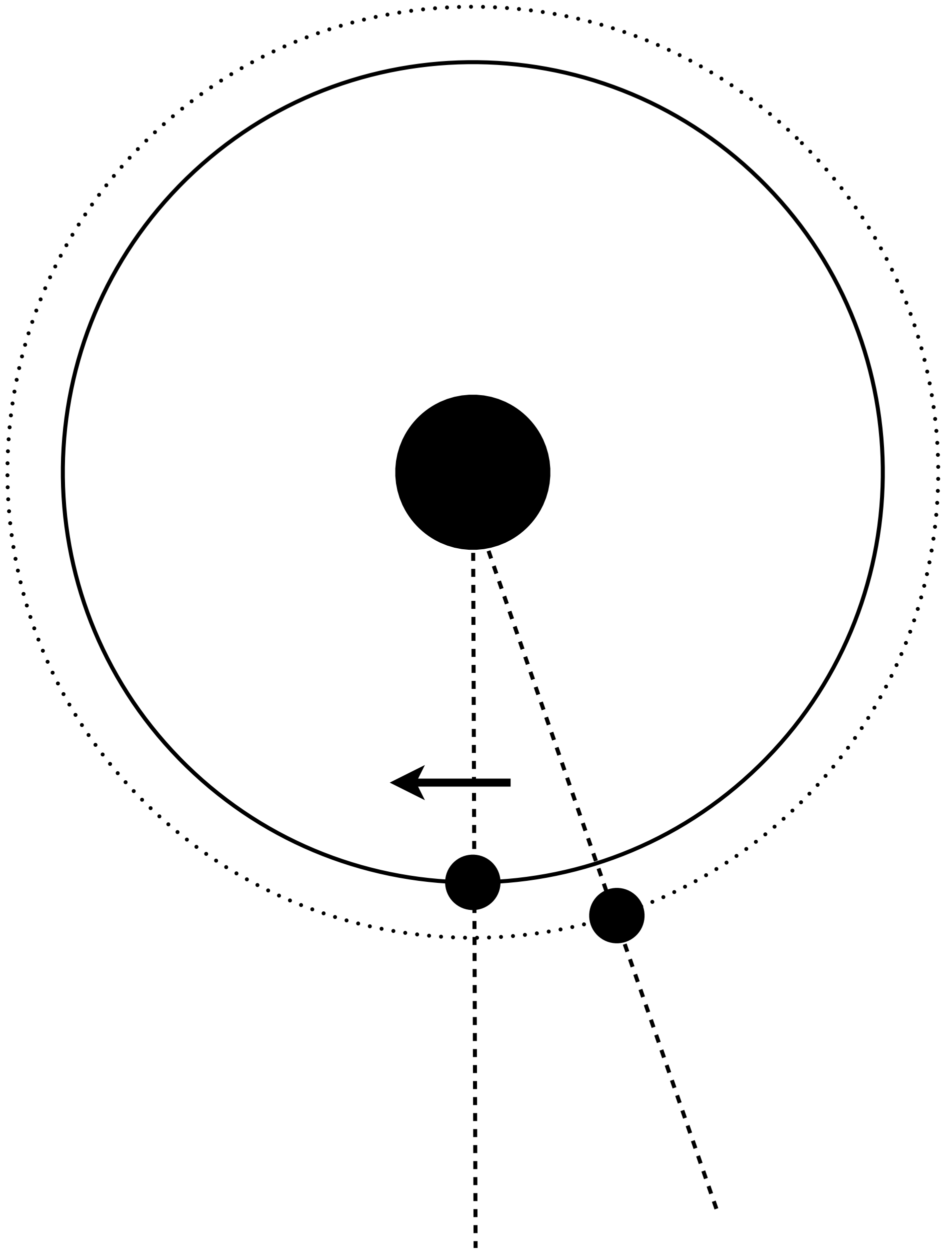
Porco et al. 2007, Sremcevic et al. 2007, Tiscareno et al. 2006

Observational evidence of non-Keplerian motion



Tiscareno et al. 2010

Longitude residual



Mean motion [rad/s]

$$n = \sqrt{\frac{GM}{a^3}}$$

Mean longitude [rad]

$$\lambda = n\,t$$

$$\lambda(t) - \lambda_0(t) = \int_0^t (n_0 + n'(t'))\,dt' - \underbrace{\int_0^t n_0\,dt'}_{n_0 t}$$

Keplerian rotation: linear

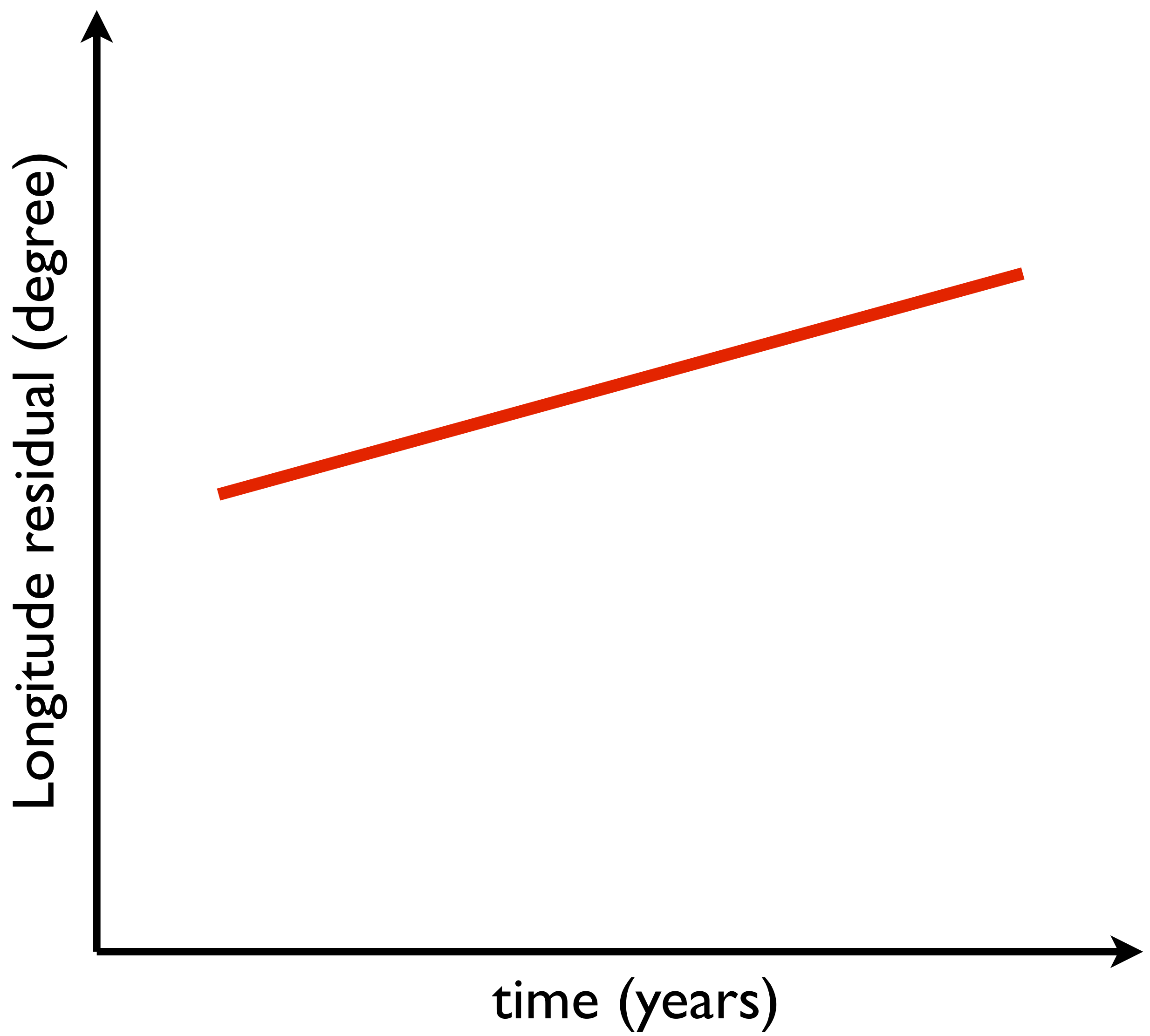
$$n'(t) = const$$

$$\lambda(t) - \lambda_0(t)$$

$$= \int_0^t (n_0 + n'(t'))\, dt'$$

$$- \int_0^t n_0\, dt'$$

$$= n_0\, t + n'\, t - n_0\, t = n'\, t$$



Constant migration rate: quadratic

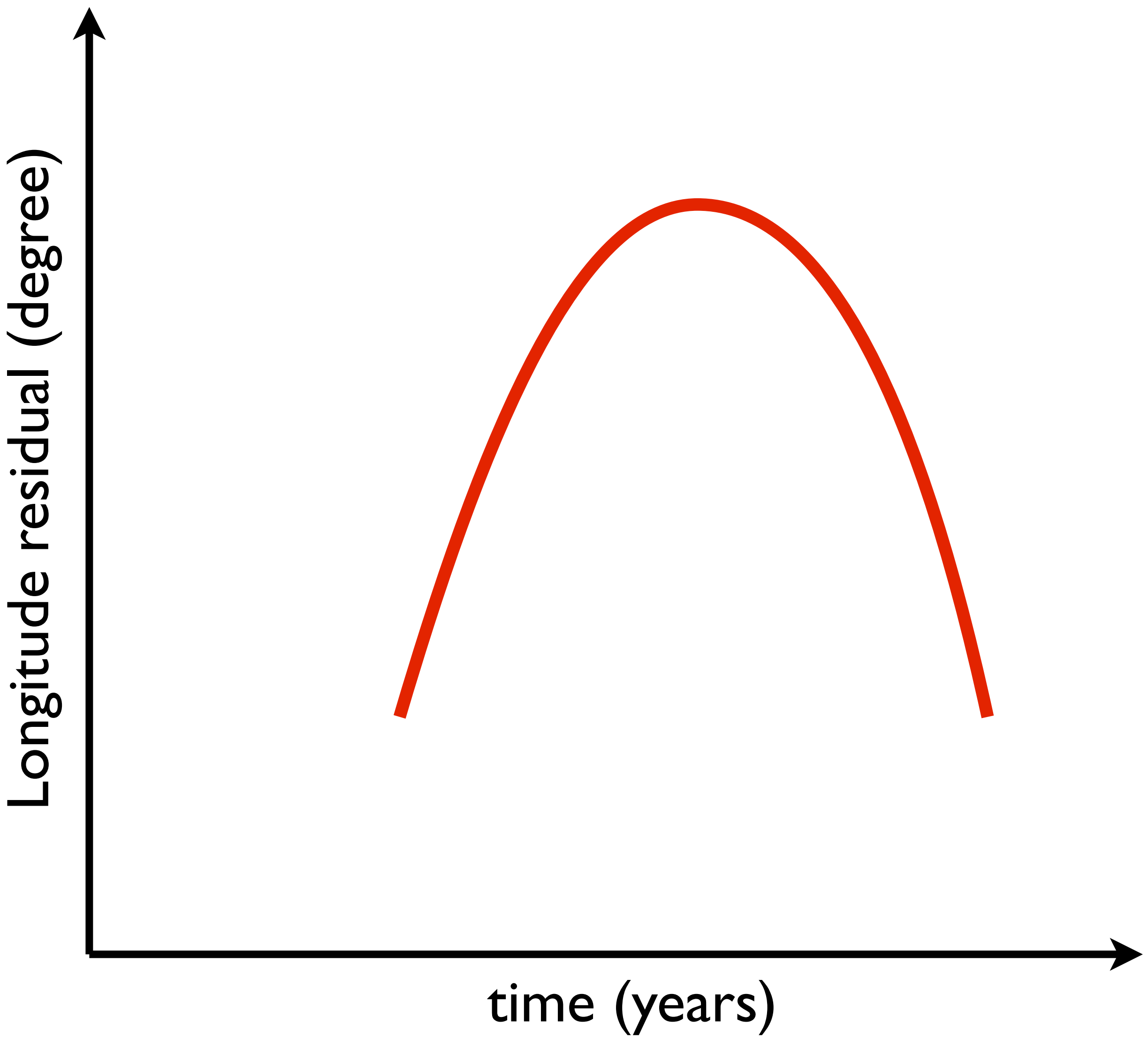
$$n'(t) = const \cdot t$$

$$\lambda(t) - \lambda_0(t)$$

$$= \int_0^t (n_0 + n'(t'))\, dt'$$

$$- \int_0^t n_0\, dt'$$

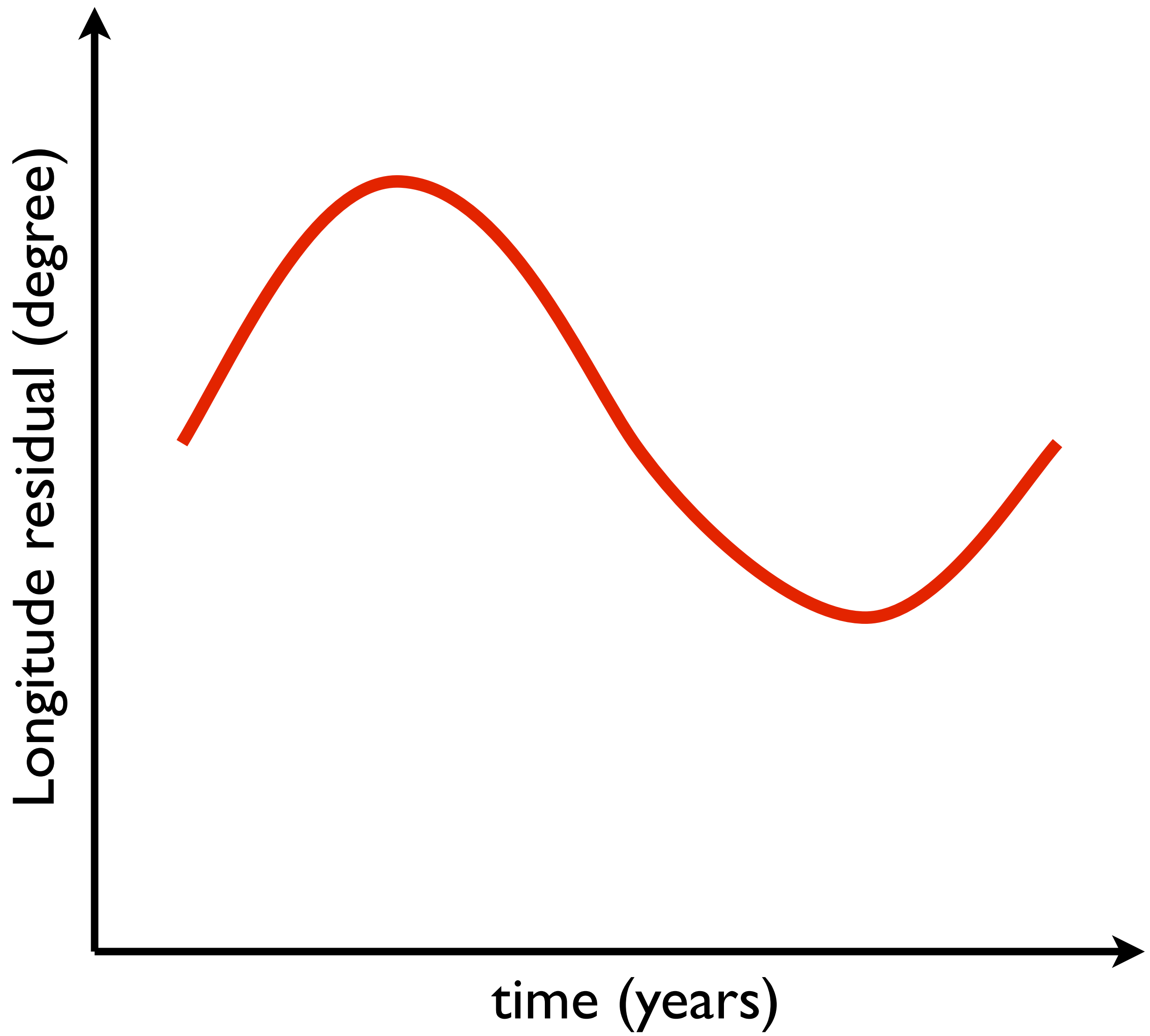
$$= \frac{1}{2} const \cdot t^2$$



Resonance: sine-curve

$$n'(t) = cos(t)$$

$$\lambda(t) - \lambda_0(t)$$
$$= \int_0^t (n_0 + n'(t'))\, dt'$$
$$- \int_0^t n_0\, dt'$$
$$= \sin(t)$$



Random walk

$$n'(t) = \int_0^t F(t')\,dt'$$

stochastic force

$$\langle F(t) \rangle = 0$$

$$\langle F(t)F(t+\Delta t) \rangle = \left\langle F^2 \right\rangle e^{-\Delta t/\tau_c}$$

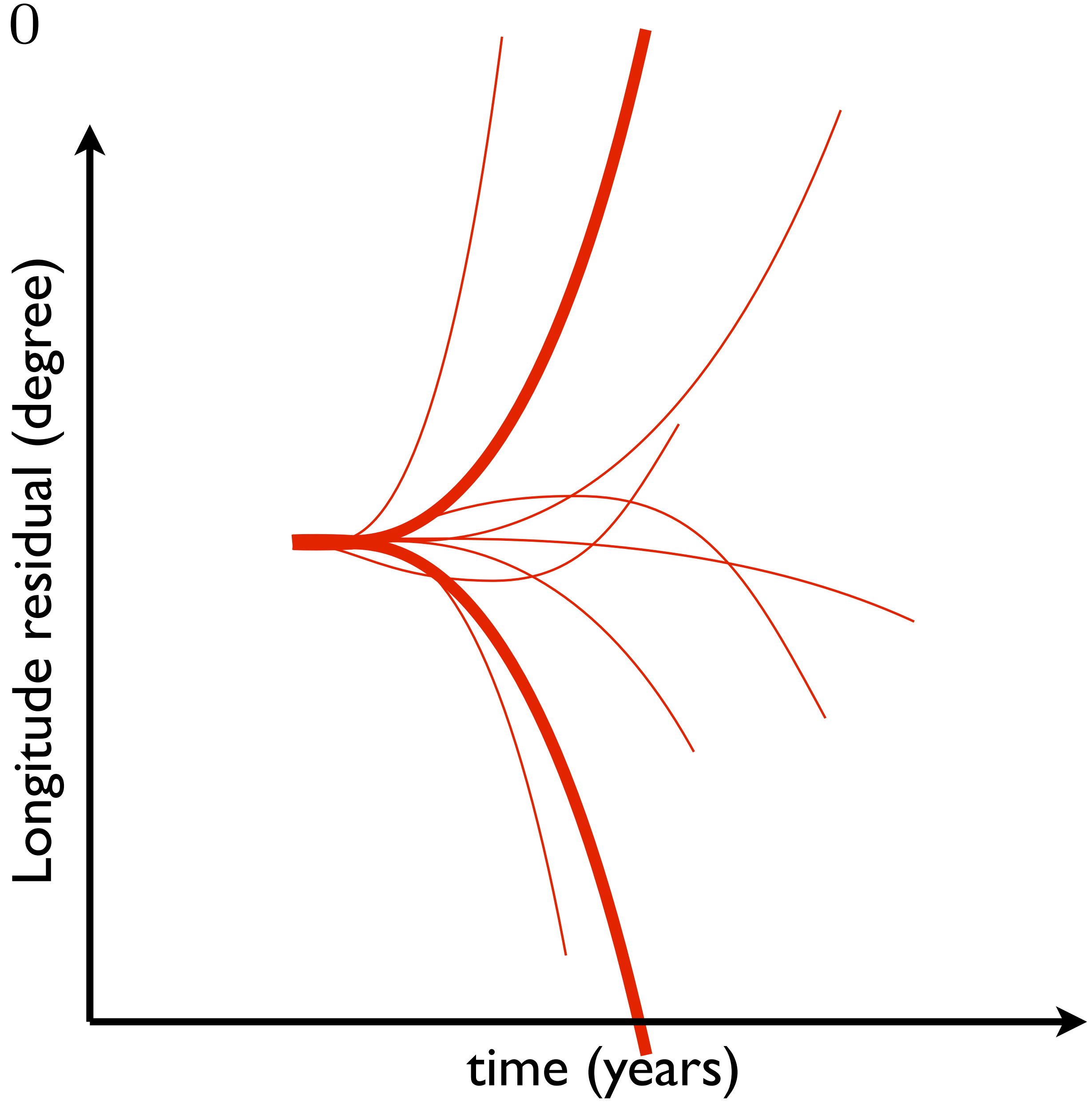
$$\left\langle (\lambda(t) - \lambda_0(t))^2 \right\rangle$$

$$= \iiiint_0^{t,t',t,t'''} F(t'')F(t'''')\,dt''''\,dt'''\,dt''\,dt'$$

$$= \langle F^2 \rangle \left(-2\tau^4 + \left(2\tau^3 t + 2\tau^4 + \tau^2 t^2\right) e^{-t/\tau} + \frac{1}{3}\tau t^3 \right)$$

Random walk

$$n'(t) = \int_0^t F(t')\,dt'$$

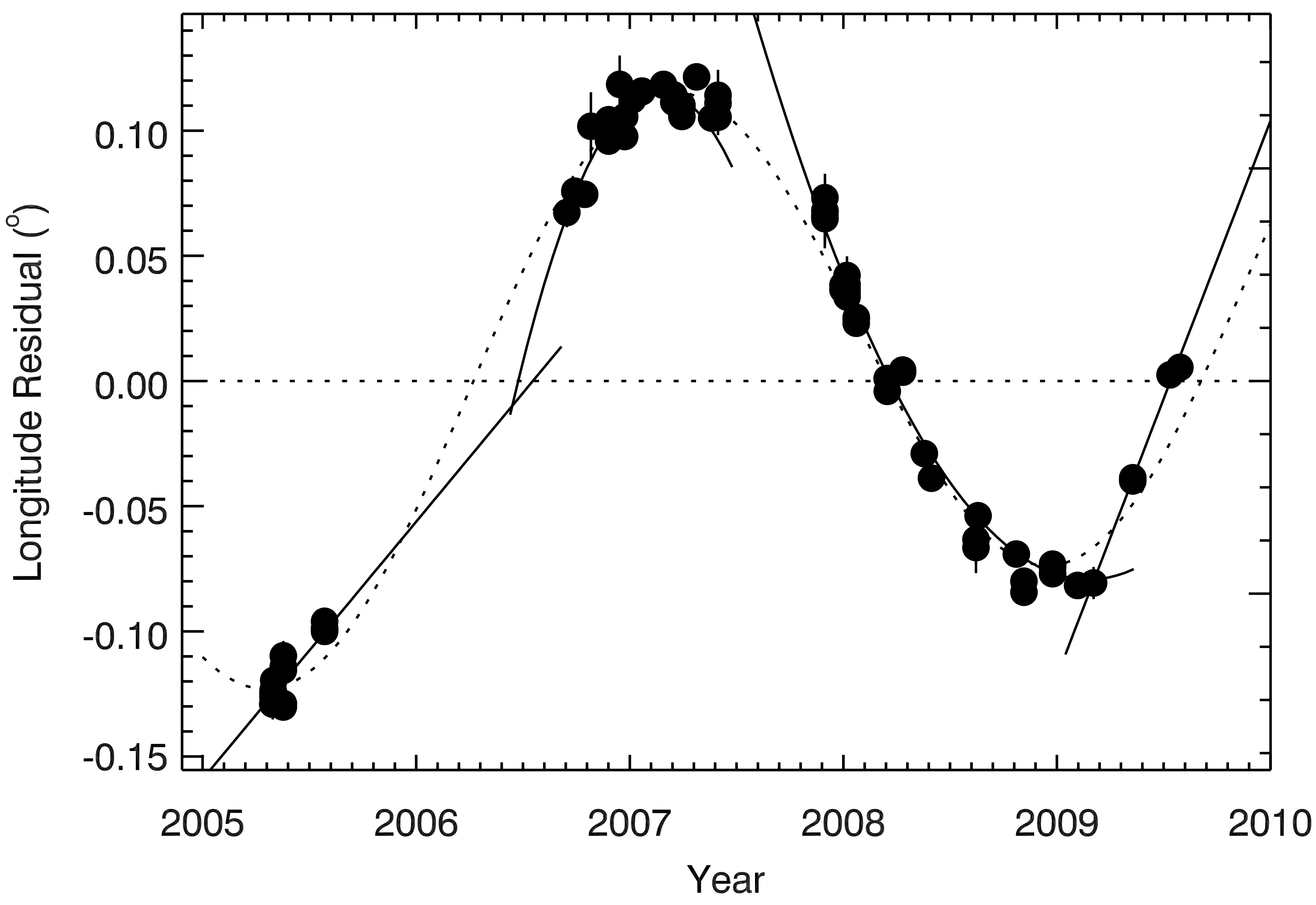


$$|\lambda(t) - \lambda_0(t)|$$

$$= \sqrt{\frac{\langle F^2 \rangle}{\tau}} t^{3/2}$$

On average!

Observational evidence of non-Keplerian motion



Tiscareno et al. 2010

Moonlets in Saturn's Rings
II. Explanations for non-Keplerian motion

Resonance with a moon

PRO

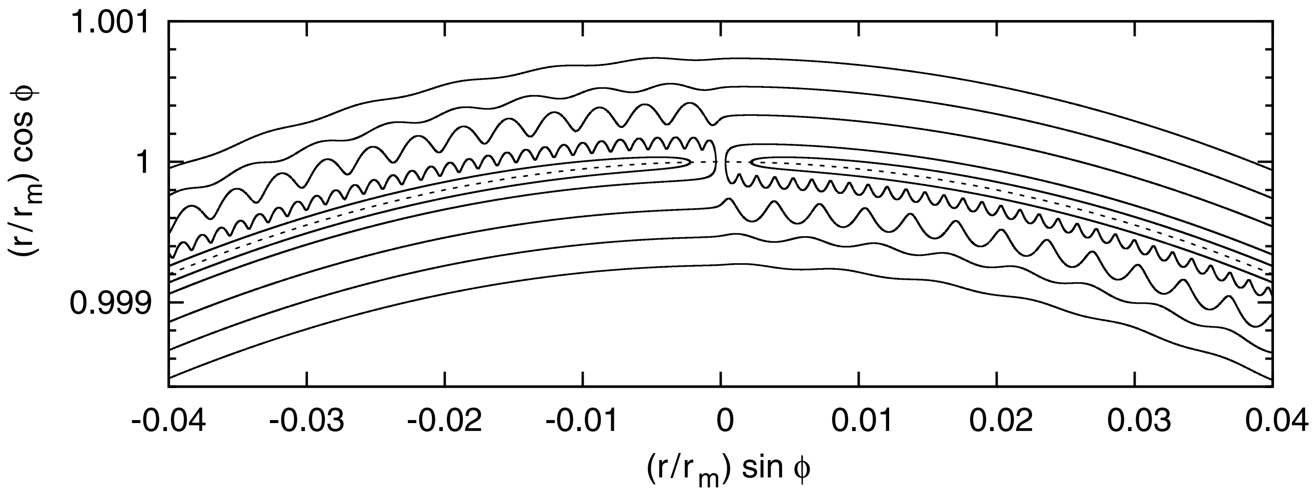
- Produces sine-shaped residual longitude
- Amplitude is a free parameter

CONTRA

- No resonance found
- Cannot fully explain shape of observations
- Other moonlets seem to migrate as well

Modified Type I migration

- Due to curvature (would be zero in shearing sheet)
- Similar to planetary migration in a gas disc
- No gas pressure
- Migration rate can be calculated analytically



$$\frac{dr_m}{dt} = -35.6\,\frac{\Sigma {r_m}^2}{M}\left(\frac{m}{M}\right)^{1/3} r_m\Omega.$$

Modified Type I migration

PRO

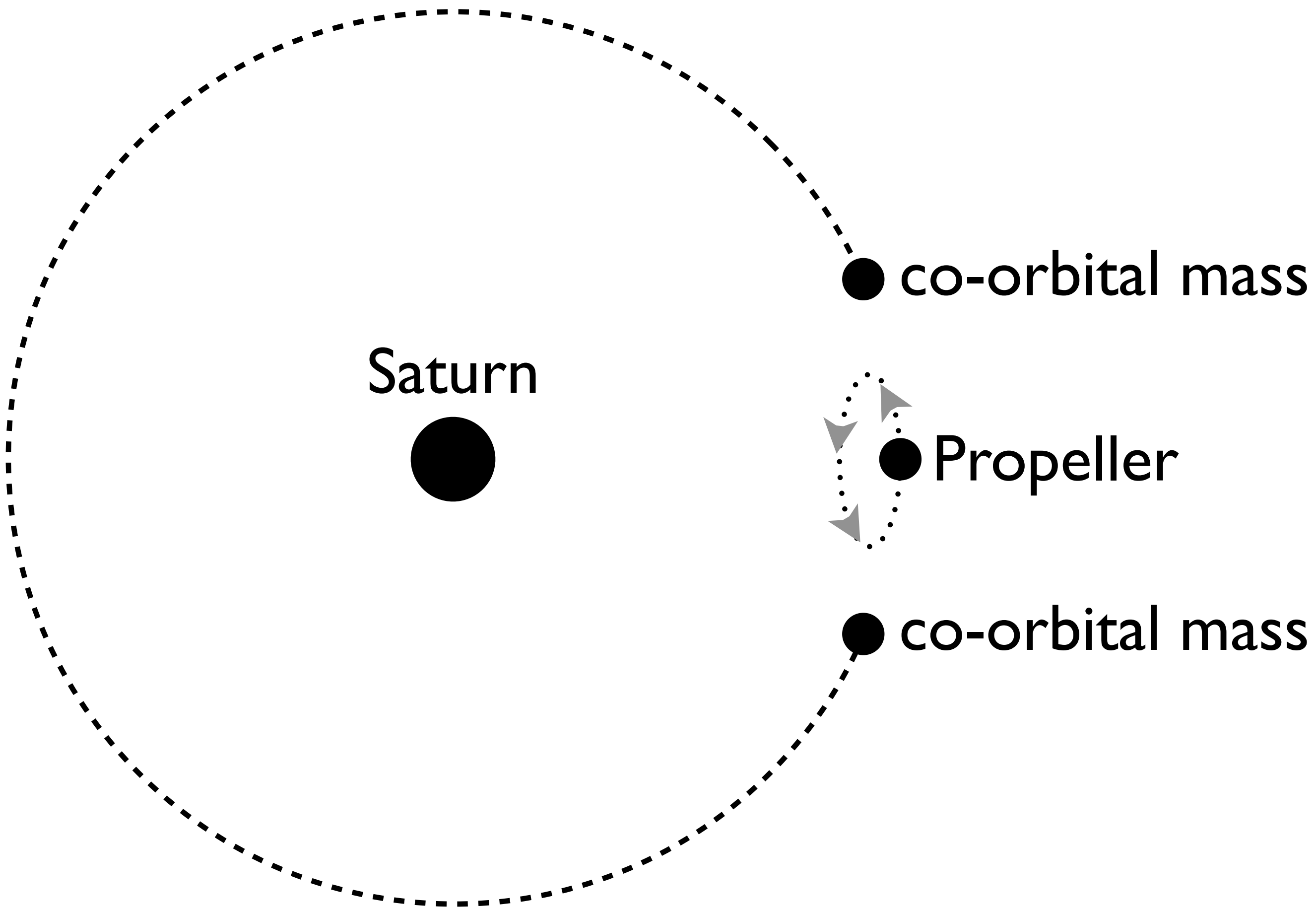
- Robust
- Would be a direct observation of type I migration

CONTRA

- Tiny migration rate ~20 cm/year
- Cannot explain shape of observations

Crida et al. 2010

Frog resonance



Frog resonance

PRO

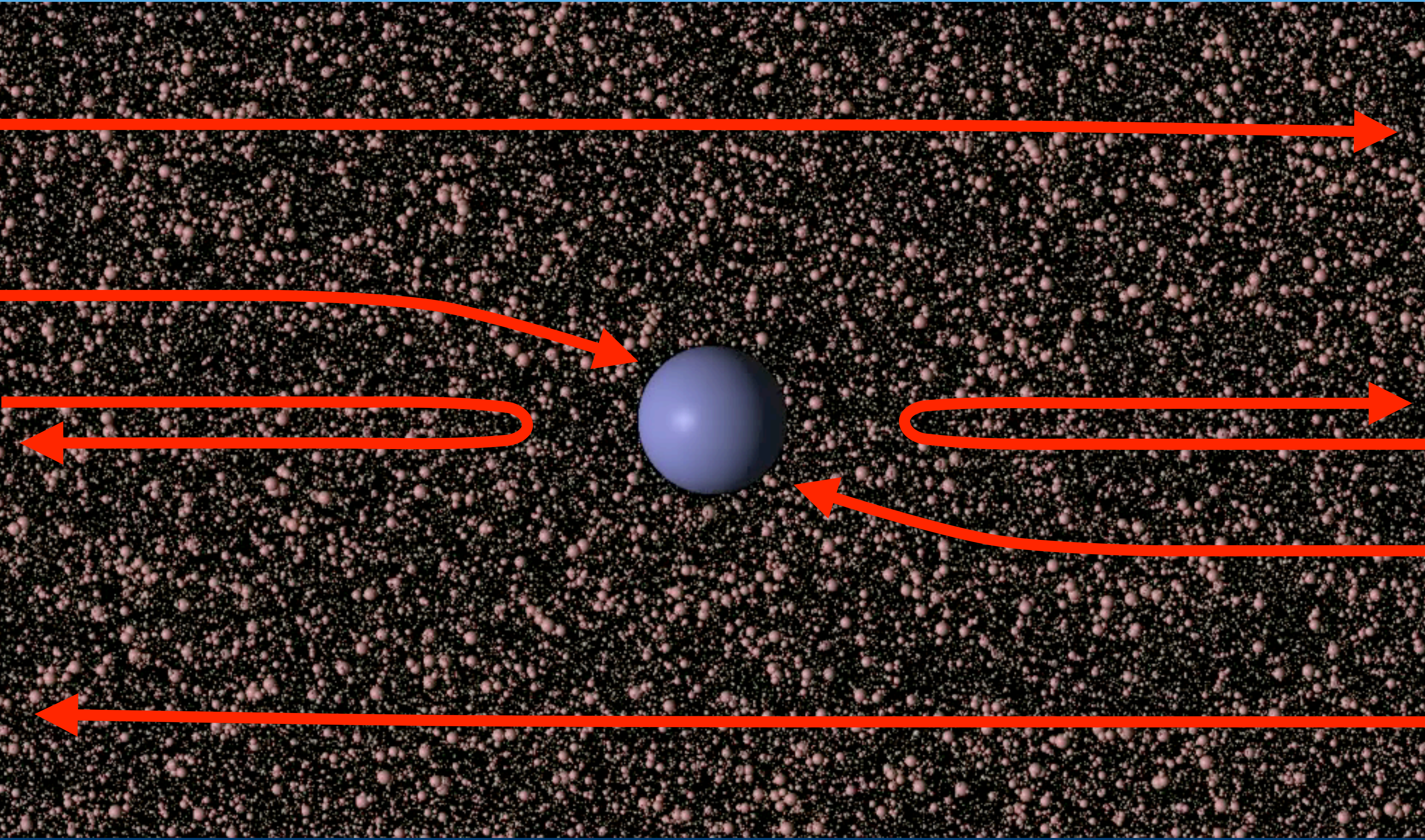
- Predicts largest period very well
- Amplitude is a free parameter

CONTRA

- Unclear if density distribution is like in the toy model
- Cannot fully explain shape of observations

Pan & Chiang 2010

Random walk



Rein & Papaloizou 2010, Crida et al 2010

Two different approaches

Analytic model

Describing evolution in a statistical manner

Partly based on Rein & Papaloizou 2009

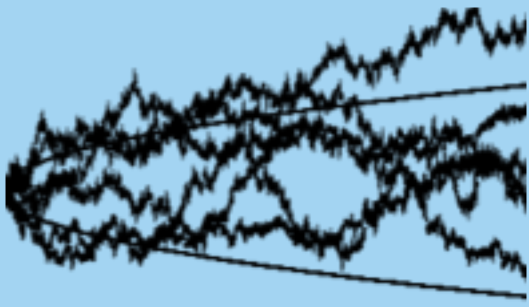
$$\Delta a = \sqrt{4\frac{Dt}{n^2}}$$

$$\Delta e = \sqrt{2.5\frac{\gamma Dt}{n^2 a^2}}$$

N-body simulations

Measuring random forces or integrating moonlet directly

Crida et al 2010, Rein & Papaloizou 2010



Effects contributing to the eccentricity evolution

- Laminar collisions
- Particles colliding
- Laminar horseshoe
- Laminar circulating
- Particles circulating
- Clumps circulating

→ **Equilibrium eccentricity**

Damping | Excitation

Rein & Papaloizou 2010, Crida et al 2010

... semi-major axis evolution

Particles colliding

Particles horseshoe

Particles circulating

Clumps circulating

Damping

Excitation

→

Random walk in semi-major axis

+Net "Type I" migration

Rein & Papaloizou 2010, Crida et al 2010

Random walk

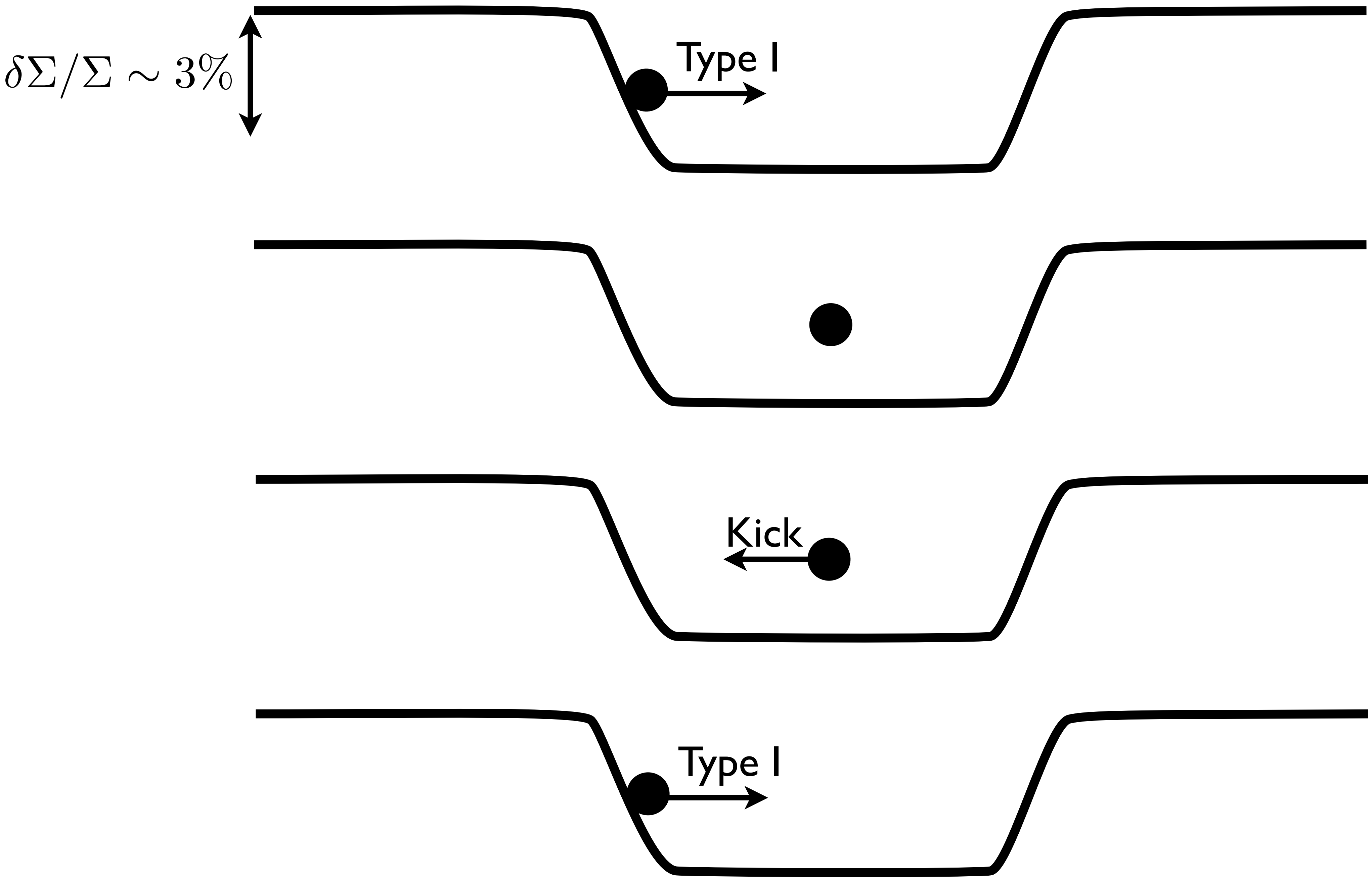
PRO

- Can explain the shape of the observations very well

CONTRA

- Has only been tested numerically for small moonlets
- No metric to test how good it matches the observations

Hybrid Type I migration / stochastic kicks



Tiscareno (in prep)

Hybrid Type I migration / stochastic kicks

PRO

- Can explain all observations very well

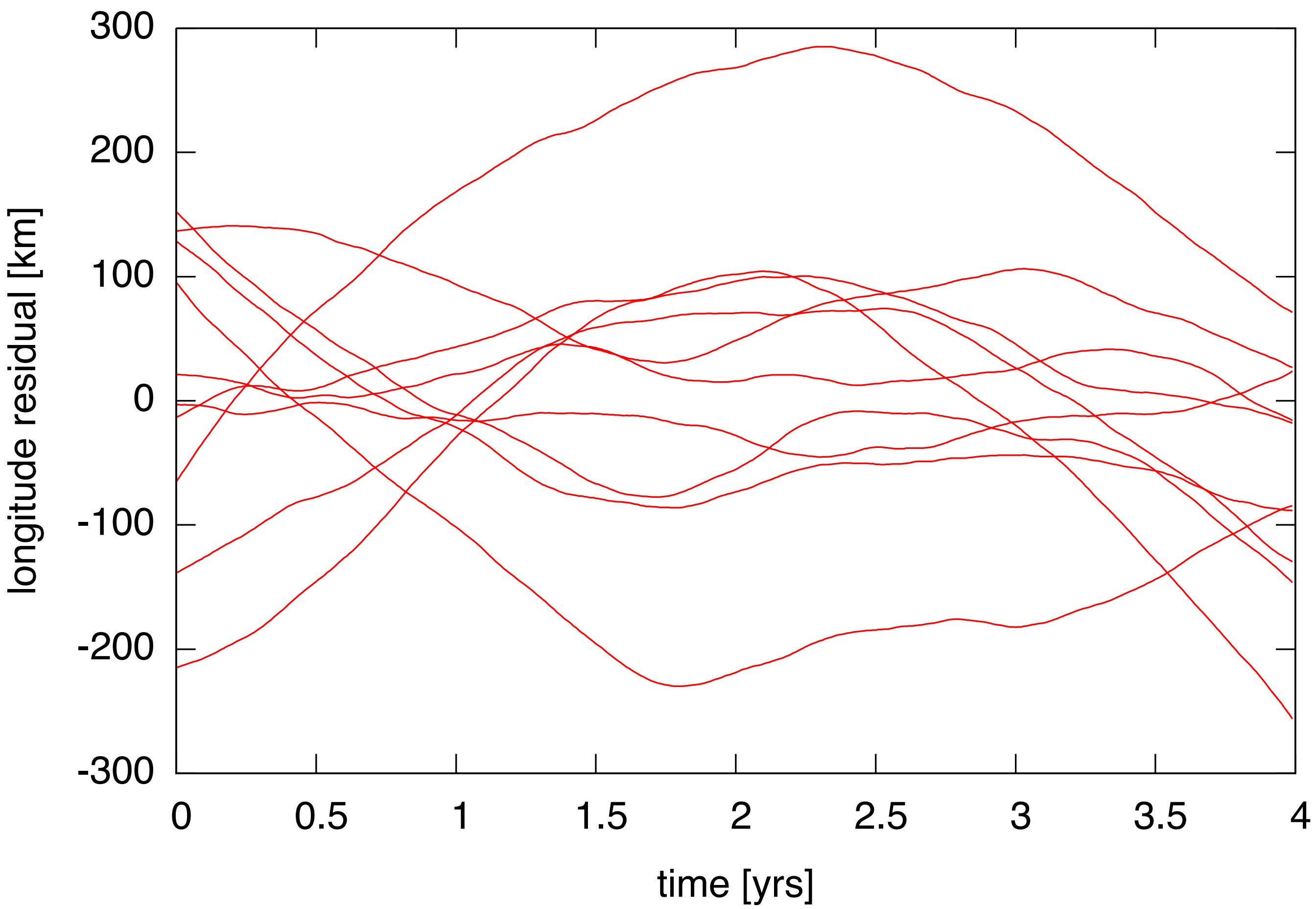
CONTRA

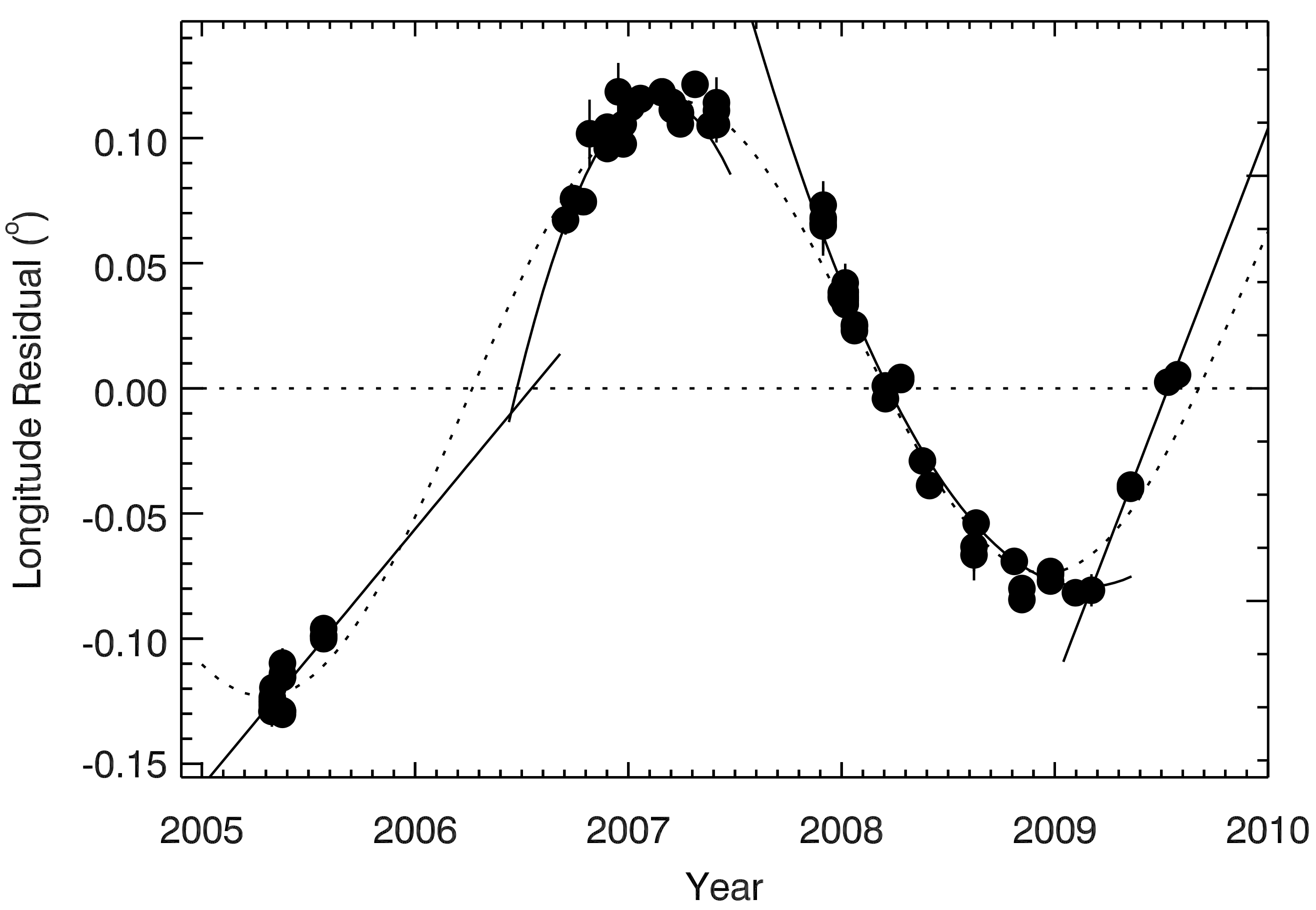
- Many free parameters: surface density profile, kicks

- Needs large kicks
 (maybe not)

Tiscareno (in prep)

Need a metric





Conclusions

Conclusions

Resonances and multi-planetary systems

- Multi-planetary system provide insight in otherwise unobservable formation phase
- Overwhelming evidence that dissipative effects (disc) shaped many systems
- Turbulence can be traced by observing orbits of multi-planetary systems
- Need precise orbital parameters to do that
- Kepler data is not good enough
- Distinctive from non-turbulent migration scenarios, clear signal
- HD45364 formed in a massive disc

Moonlets in Saturn’s rings

- Small scale version of the proto-planetary disc
- Dynamical evolution can be directly observed
- Evolution is most likely dominated by random-walk
- Caused by collisions and gravitational wakes
- Might lead to independent age estimate of the ring system

REBOUND

A new open source collisional N-body code

http://github.com/hannorein/rebound